EXERCISE 1.2

1.Calculate D_6 and P_{85} for the following data : 79,82,36,38,51,72,68,70,64,63

Solution :

The given data can be arranged in ascending order as follows : 36,38,51,63,64,68,70,72,79,82.

$Here, n = 10$

$$D_6 = Value\ of\ 6\left(\frac{n+1}{10}\right)^{th}\ observation$$

$$D_6 = Value\ of\ 6\left(\frac{10+1}{10}\right)^{th}\ observation$$

$$D_6 = Value\ of\ (6 \times 1.1)^{th}\ observation$$

$$D_6 = Value\ of\ (6.6)^{th}\ observation$$

$$D_6 = Value\ of\ 6^{th}\ observation +$$
$$0.6\ (Value\ of\ 7^{th} observation - value\ of\ 6^{th}\ observation)$$

$$D_6 = 68 + 0.6(70 - 68)$$

$$D_6 = 69.2$$

$$P_{85} = Value\ of\ 85\left(\frac{n+1}{100}\right)^{th}\ observation$$

$$P_{85} = Value\ of\ 85\left(\frac{10+1}{100}\right)^{th}\ observation$$

$$P_{85} = Value\ of\ (85 \times 0.11)^{th}\ observation$$

$$P_{85} = Value\ of\ (9.35)^{th}\ observation$$

$$P_{85} = Value\ of\ 9^{th}\ observation +$$
$$0.35\ (Value\ of\ 10^{th} observation$$

2.The daily wages (in) of 15 laboures are as follows :
230,400,350,200,250,380,210,225,375,180, 375,450,300,350,250 Calculate D_8 and P_{90}

Solution :
The given data can be arranged in ascending order as follows : 180, 200, 210, 225, 230, 250, 250, 300, 350, 350, 375, 375, 380, 400, 450.

$Here, n = 10$

$$D_8 = Value\ of\ 8\left(\frac{n+1}{10}\right)^{th}\ observation$$

$$D_8 = Value\ of\ 8\left(\frac{15+1}{10}\right)^{th}\ observation$$

$$D_8 = Value\ of\ (8 \times 1.6)^{th}\ observation$$

$$D_8 = Value\ of\ (12.8)^{th}\ observation$$

$$D_8 = Value\ of\ 12^{th}\ observation +$$
$$0.8\ (Value\ of\ 13^{th} observation - value\ of\ 12^{th}\ observation)$$

$$D_8 = 375 + 0.8(380 - 375)$$

$$D_8 = 375 + 0.8(5) = 375 + 4 = 379$$

$$P_{90} = Value\ of\ 90\left(\frac{n+1}{10}\right)^{th}\ observation$$

$$P_{90} = Value\ of\ 90\left(\frac{15+1}{10}\right)^{th}\ observation$$

$$P_{90} = Value\ of\ (90 \times 0.16)^{th}\ observation$$
$$= Value\ of\ (14.4)^{th}\ observation$$
$$= Value\ of\ 14^{th}\ observation +$$
$$0.4\ (Value\ of\ 15^{th} observation$$

3.Calculate 2nd decile and 65th percentile for the following :

x	80	100	120	145	200	280	310	380	400	410
f	15	18	25	27	40	25	19	16	8	7

x	f	Less than cumulative frequency (c.f)
80	15	15
100	18	33
120	25	58 ← D_2
145	27	85
200	40	125
280	25	150 ← P_{65}
310	19	169
380	16	185
400	8	193
410	7	200
Total	*200*	

Solution :

$Here, n = 10$

$$D_2 = Value\ of\ 2\left(\frac{n+1}{10}\right)^{th}\ observation$$

$$D_2 = Value\ of\ 2\left(\frac{200+1}{10}\right)^{th}\ observation$$

$$D_2 = Value\ of\ (8 \times 20.1)^{th}\ observation$$

$$D_2 = Value\ of\ (40.2)^{th}\ observation$$

Cumulative frequency which is just greater than (or equal to) 40.2 is 58.

$\therefore D_2 = 120$

$$P_{65} = Value\ of\ 65\left(\frac{n+1}{10}\right)^{th}\ observation$$

$$P_{65} = Value\ of\ 65\left(\frac{200+1}{100}\right)^{th}\ observation$$

$$P_{65} = Value\ of\ (65 \times 2.01)^{th}\ observation$$

$$P_{65} = value\ of\ (130.65)^{th}\ observation$$

$Cumulative\ frequency\ which\ is\ just\ greater\ than\ (or\ equal\ to)$ 130.65 is 150

$P_{65} = 280$

4.From the following data calculate the rent of 15th, 65th & 91st house.

House Rent (in)	11000	12000	13000	15000	14000	16000	17000	18000
No. of houses	25	17	13	14	15	8	6	2

House Rent (Rs)	*No of Houses*	**Less than cumulative frequency (c.f)**
11,000	25	25 $\leftarrow P_{15}$
12,000	17	42
13,000	13	55
14,000	14	70 $\leftarrow P_{65}$
15,000	15	84
16,000	8	92
17,000	6	98
18,000	2	100
Total	*100*	

Solution :$Here, n = 10$

$$P_{15} = Value\ of\ 15\left(\frac{n+1}{100}\right)^{th}\ observation$$

$$P_{15} = Value\ of\ 15\left(\frac{100+1}{100}\right)^{th}\ observation$$

$$P_{15} = Value\ of\ (15 \times 1.01)^{th}\ observation$$

$$P_{15} = Value\ of\ (15.15)^{th}\ observation$$

$Cumulative\ frequency\ which\ is\ just\ greater\ than\ (or\ equal\ to)$ $15.65\ is\ 25$

$$P_{15} = 11{,}000$$

$$P_{65} = Value\ of\ 65\left(\frac{n+1}{100}\right)^{th}\ observation$$

$$P_{15} = Value\ of\ 65\left(\frac{100+1}{100}\right)^{th}\ observation$$

$$P_{15} = Value\ of\ (65 \times 1.01)^{th}\ observation$$

$$P_{15} = Value\ of\ (65.65)^{th}\ observation$$

$Cumulative\ frequency\ which\ is\ just\ greater\ than\ (or\ equal\ to)$ $65.65\ is\ 70$

$$P_{65} = 14{,}000$$

$$P_{92} = Value\ of\ 92\left(\frac{n+1}{100}\right)^{th}\ observation$$

$$P_{92} = Value\ of\ 92\left(\frac{100+1}{100}\right)^{th}\ observation$$

$$P_{92} = Value\ of\ (92 \times 1.01)^{th}\ observation$$

$$P_{92} = Value\ of\ (92.92)^{th}\ observation$$

$Cumulative\ frequency\ which\ is\ just\ greater\ than\ (or\ equal\ to)\ 92.92\ is\ 98$

$\therefore\ P_{92} = 17{,}000$

5.The following frequency distribution shows the weight of students in a class.
i.)Find the percentage of students whose weight is more than 50 kg.
ii.)If the weight column provided is of mid values then find the percentage of students whose weight is more than 50 kg.

Weight (in Kg)	40	45	50	55	60	65
Number of Students	15	40	29	21	10	5

Solution:
i.)

Weight (in Kg)	Number of Students	Less than cumulative frequency (c.f)
40	15	15
45	40	55
50	29	84
55	21	105
60	10	115
65	5	120
Total	120	

Let the percentage of student weighting less than $50\ kg$ *be* x

$\therefore P_x = 50$

From the table, out of 20 *students,* 84 *students have their weight less than* $50\ kg$

$\therefore$ *Number df student weighting more than* $50\ kg$

$= 120 - 84 = 36$

$\therefore$ *percentage of student having there weight more than* $50\ kg = \frac{36}{120} \times 100 = 30\%.$

Solution:

ii.) The difference between any consecutive mid values of weight is 5 kg The class intervals must of width 5, with 40, 45, ... as their md values.

∴ The class intervals will be 37.5 − 42.5, 42.5 − 47.5, etc.

We consrut the less than cumulative frequency table as given below:

Weight (in Kg)	Number of Students	Less than cumulative frequency (c.f)
37.5- 42.5	15	15
42.5-47.5	40	55
47.5-52.5	29	84
52.5-57.5	21	105
57.5-62.5	10	115
62.5-67.5	5	120
Total	120	

Here, $N = 120$

Let $P_x = 50$

The value of 50 *lies in the class* $47.5 - 52.5$.

$\therefore L = 47.5, h = 5, f = 29, c.f. = 55$

$$P_x = L + \frac{h}{f}\left(\frac{xN}{100} - c.f.\right)$$

$$\therefore 50 = 47.5 + \frac{5}{29}\left(\frac{x \times 120}{100} - 55\right)$$

$$\therefore 50 - 47.5 = \frac{5}{29}\left(\frac{6x}{5} - 55\right)$$

$$\therefore 2.5 = \frac{5}{29}\left(\frac{6x}{5} - 55\right)$$

$$\therefore \frac{6x}{5} - 55 = 2.5 \times \frac{29}{5}$$

$$\therefore \frac{6x}{5} - 55 = 14.5$$

$$\therefore \frac{6x}{5} = 55 + 14.5$$

$$\therefore \frac{6x}{5} = 69.5$$

$$\therefore x = 69.5 \times \frac{5}{6}$$

$\therefore x = 58$ *(approximately)*

∴ 58% of students are having weight below $50kg$.

∴ Percentage of student having bweight above $50kg$ *is* $100 - 58 = 42$

∴ 42% of students are having weight above 50kg.

6.Calculate D_4 and P_{48} from the following data:

Mid Value	2.5	7.5	12.5	17.5	22.5	Total
Frequency	7	18	25	30	20	100

Solution:

The difference between any two consecuutive mid valueue is 5, *widht of the class intterval* $= 5$

$\therefore$ *Class interval with mid value* 2.5 *is* $i0 - 5$

Class interval with mid value 7.5 *is* $5 - 10$, *etc.*

We constuct the less than cumulative frequency table.

Class interval	Frequency (f)	Less than cumulative frequency (c.f)
0-5	7	7
5-10	18	25
10-15	25	50 $\leftarrow D_4$ P_{48}
15-20	30	80
20-25	20	100
Total	100	

$Here, N = 100$

$$D_4 class = class\ containing \left(\frac{4N}{10}\right)^{th}\ observation$$

$$\therefore \frac{4N}{10} = \frac{4 \times 100}{10} = 40.$$

Cumulaative frequency which is greater than (or equal to)40 *is* 50.

$$\therefore \quad D_4 = L + \frac{h}{f}\left(\frac{4N}{10} - c.f.\right)$$

$$= 10 + \frac{5}{25}(40 - 25)$$

$$= 10 + \frac{1}{5}(15)$$

$$= 10 + 3$$

$$\therefore \quad D_4 = 13$$

$$P_{48} class = class\ containing \left(\frac{48N}{100}\right)^{th}\ observation$$

$$\therefore \left(\frac{48N}{100}\right) = \frac{48 \times 100}{100} = 48$$

Cumulaative frequency which is greater than (or equal to)40 *is* 50.

$\therefore P_{48}$ *lies in the class* $10 - 15$

$\therefore L = 10, h = 5, f = 25, c.f. = 25$

$$\therefore \quad P_{48} = L + \frac{h}{f}\left(\frac{48N}{100} - c.f.\right)$$

$$= 10 + \frac{5}{25}(48 - 25)$$

$$= 10 + \frac{1}{5}(23)$$

$$= 10 + 4.6$$

$$\therefore \quad P_{48} = 14.6$$

7.Calculate *D9* and *P20* of the following distribution.

Length (in Inches)	0-20	20-40	40-60	60-80	80-100	100-120
No. of Units	1	14	35	85	90	15

Solution :We Construct the less than cumulative frequency table as given below :

Length (in Inches)	No. of units	Less than cumulative frequency (c.f)
00-20	1	1
20-40	14	15
40-60	35	50 $\leftarrow P_{20}$
60-80	85	135
80-100	90	225 $\leftarrow D_9$
100-120	15	240
Total	240	

Here, N=240,

$$D_9\ Class = class\ containing \left(\frac{9N}{10}\right)^{th}\ Observation$$

$$\therefore \frac{9N}{10} = \frac{9\times240}{10} = 216$$

Cumulative frequency which is just greater than (or equal to) 216 is 225

$\therefore D_9\ lies\ in\ the\ class\ 80-100.$

$\therefore L = 80, h = 20, f = 90, c.f = 135$

$$\therefore D_9 = L + \frac{h}{f}\left(\frac{9N}{10} - c.f\right) \qquad = 80 + \frac{20}{90}(216 - 135)$$

$$= 80 + \frac{2}{9}(81)$$

$$D_9 = 80 + 18$$

$$D_9 = 98$$

$$P_{20}\ class = class\ containing \left(\frac{20N}{100}\right)^{th}\ observation$$

$$\therefore \frac{20N}{100} = \frac{20\times240}{100} = 48$$

Cumulative frequency which is just greater than (or equal to) 48 is 50.

$\therefore L = 40, h = 20, f = 35, c.f = 15$

$$\therefore P_{20} = L + \frac{h}{f}\left(\frac{20N}{100} - c.f\right) \qquad = 40 + \frac{20}{35}(48 - 15)$$

$$= 40 + \frac{4}{7}(33)$$

$$= 40 + \frac{132}{7} \qquad = 40 + 18.86$$

$$\therefore P_{20} = 58.86$$

8.Weekly wages for group of 100 persons are given below :

Wages (in)	0-500	500-1000	1000-1500	1500-2000	2000-2500
No. of persons	7	?	25	30	?

D3 for this group is 1100 Calculate the missing frequencies.

Solution:

Let a and b be the missing frquencies of the class 500 − 1000 *and class* 2000 − 2500 *respectively.*

Class interval	No. of persons	Less then cumulative frquency
0-500	7	7
500-1000	a	7+a
1000-1500	25	32+a ← D_3
1500-2000	30	62+a
2000-2500	b	62+a+b
Total	62+a+b	

Here, $N = 62 + a + b$ Since, $N = 100$.

$\therefore 62 + a + b = 100$

$\therefore a + b = 100 \quad \ldots (i)$

Given, $D_3 = 1100$

$\therefore D_3$ *lies in the class* $1000 - 1500$.

$\therefore L = 100, h = 500, f = 25, \frac{3N}{10} = \frac{3\times100}{10} = 30, c.f. = 7 + a$

$$\therefore D_3 = L + \frac{h}{f}\left(\frac{3N}{10} - c.f\right)$$

$$\therefore 1100 = 1000 + \frac{500}{25}[(30 - (7 + a)]$$

$$\therefore 1100 - 1000 = 20(30 - 7 - a)$$

$$\therefore 100 = 20(23 - a)$$

$$\therefore 20a = 460 - 100$$

$$\therefore a = \frac{360}{20}$$

$\therefore a = 18.$

Substituting the value of a in equation (i), *we get*

$18 + b = 38$

$\therefore b = 38 - 18 = 20$

$\therefore$ 18 *and* 20 *are the missing frequencies of the class* 500 − 1000 *and* 2000 − 2500 *respectively.*

9.The weekly profit (in rupees) of 100 shops are distributed as follows :

Profit per shop	No. of Shops
0-1000	10
1000-2000	16
2000-3000	26
3000-4000	20
4000-5000	20
5000-6000	5
6000-7000	3

Find the limits of the profit of middle 60% of the shops.
We construct the less then cumulative frequency tables as given below:

Profit Per Shop (in Rs)	No. Of Shop (f)	Less Then Cumulative Frequency (c.f.)
0-1000	10	10
1000-2000	16	26 $\leftarrow P_{20}$
2000-3000	26	52
3000-4000	20	72
4000-5000	20	92 $\leftarrow P_{80}$
5000-6000	5	97
6000-7000	3	100
Total	100	

$Here, N = 100$

$P_{20}\ class = class\ containing\ \left(\frac{20N}{100}\right)^{th}\ observation$

$\therefore \frac{20N}{100} = \frac{20 \times 100}{100} = 20$

$Cumulative\ frequency\ which\ is\ greter\ (or\ equal)\ 20\ is\ 26$

$\therefore P_{20}\ lies\ in\ the\ class\ 1000 - 2000.$

$\therefore L = 1000, h = 1000, f = 16\ c.f = 1$

$\therefore P_{20} = L + \frac{h}{f}\left(\frac{20N}{100} - c.f\right) = 1000 + \frac{1000}{16}(20 - 10)$

$= 1000 + \frac{125}{2}(10) = 1000 + 625$

$\therefore P_{20} = 1625$

$P_{80}\ class = class\ containing\ \left(\frac{80N}{100}\right)^{th}\ observation$

$\therefore \frac{80N}{100} = \frac{80 \times 100}{100} = 80$

$Cumulative\ frequency\ which\ is\ greter\ (or\ equal)\ 80\ is\ 92$

$\therefore P_{80}\ lies\ in\ the\ class\ 4000 - 5000.$

$\therefore L = 4000, h = 1000, f = 20\ c.f = 72$

$\therefore P_{20} = L + \frac{h}{f}\left(\frac{80N}{100} - c.f\right) = 4000 + \frac{1000}{20}(80 - 72)$

$= 8000 + 50(8) = 4000 + 400$

$\therefore P_{20} = 4400$

$\therefore the\ profit\ of\ middle\ 60\%\ of\ shops\ lie\ between\ the\ limits\ Rs\ 1{,}625\ to\ Rs\ 4{,}400.$

10.In a particular factory, workers produce various types of output units The following distribution was obtained.

Outputs units Produced	No. of workers
70-74	40
75-79	45
80-84	50
85-89	60
90-94	70
95-99	80
100-104	100

Find the percentage of workers who have produced less than 82 output units.

Solotion: We construct the less than cumulative frequency table as given below:

Output units produced	No. Of Workers(f)	Less Then Cumulative Frequency (c.f.)
69.5-74.5	40	40
74.5-79.5	45	85
79.5-84.5	50	135
84.5-89.5	60	195
89.5-94.5	70	265
94.5-99.5	80	345
99.5-104.5	100	445
Total	445	

Since the given data is not continuous, we have to convert it in the continuous from by substracting 0.5 from the lower limits and adding 0.5 to the upper lilmit of every class interval.

$\therefore$ *the class interval will be* $69.5-74.5, 74.5-79.5,$ *etc.*

Here, $N = 445$

Let $P_x = 82$

the value 82 lies in the class $79.5-84.5$

$\therefore L = 79.5, h = 5, f = 50\ c.f = 85$

$$\therefore P_x = L + \frac{h}{f}\left(\frac{xN}{100} - c.f\right)$$

$$\therefore 82 = 79.5 + \frac{5}{50}\left(\frac{x \times 445}{100} - 85\right)$$

$$\therefore 82 - 79.5 = \frac{1}{10}\left(\frac{89x}{20} - 85\right)$$

$$\therefore 2.5 \times 10 = \frac{89x}{20} - 85$$

$$\therefore 25 + 85 = \frac{89x}{20}$$

$$\therefore 110 = \frac{89x}{20}$$

$$\therefore x = \frac{110 \times 20}{89} = 24.72$$

$\therefore$ *24.72% of workers produced less than 82 output units.*