Complex Number

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Exercise 3.1

1) Write the conjugates of the following complex numbers

i) $3 + i$

Solution:

$3 - i$

ii) $3 - i$

Solution:

$3 + i$

iii) $-\sqrt{5} - \sqrt{7}\, i$

Solution:

$-\sqrt{5} + \sqrt{7}\, i$

iv) $-\sqrt{-5}$

Solution:

$-\sqrt{-5} = -\sqrt{5 * (-1)}$

$= -\sqrt{5} \times \sqrt{-1}$

$= \sqrt{5}\, i$

v) $5i$

Solution:

$-5i$

vi) $\sqrt{5} - i$

Solution:

$\sqrt{5} + i$

vii) $\sqrt{2} + \sqrt{3}\, i$

Solution:

$\sqrt{2} - \sqrt{3}\, i$

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(2) Express the following in the form of a + i b, a, b ∈ R, $i = \sqrt{1}$ State the value of a and b.

(i) $(1+2\,i)(-2+i)$

Solution:

$$(1+2i)(-2+i) = 1(-2+i) + 2i(-2+i)$$
$$= -2 + i - 4i + 2i^2$$
$$= -2 - 3i + 2(-1) \quad \{since\ , = -1\}$$
$$= -2 - 3i - 2$$
$$= -4 - 3i$$

Comparing it with $a + bi, we\ get\ a = -4\ and\ b = -3$

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(ii) $\frac{i(4+3\,i)}{(1-i)}$

$$\frac{i(4+3i)}{(1-i)} \times \frac{(1+i)}{(1+i)} \quad \{\text{Multiply and divide by } (1+i\ )$$

$$= \frac{(4i-3i^2)\,(1+i)}{(1)^2-(i)^2}$$

$$= \frac{(4i-3)\,(1+i)}{1+1} \quad \{since\ , i^2 = -1\}$$

$$= \frac{4i3+4i^2-3i}{2}$$

$$= \frac{i-3-4}{2} \quad \{since\ , i^2 = -1\}$$

$$= \frac{-7+i}{2}$$

$$= \frac{-7}{2} + \frac{1}{2}i$$

$$= a = -\frac{7}{2} \text{ and } b = \frac{1}{2}$$

(iii) $\frac{(2+i)}{(3-i)(1+2i)}$

$= \frac{(2+i)}{(3-i)(1+2i)}$

$= \frac{2+i}{3+6i-i-2i^2}$

$= \frac{2+i}{3+5i+2}$ $\{since\ ,\ i^2\ = -1\}$

$= \frac{2+i}{5+5i} \times \frac{5-5i}{5-5i}$ {Multiply and divide by $(5+5i\ )$}

$= \frac{10-10i+5i-5i^2}{(5)^2-(5i)^2} = \frac{10-5i+5}{25+25}$ $\{since\ ,\ i^2\ = -1\}$

$= \frac{15-5i}{50}$

$= \frac{5(3-i)}{50}$

$= \frac{(3-i\ )}{10} = \frac{3}{10} - \frac{1}{10}i$

$\therefore$ a $= \frac{3}{10}$ and b $= -\frac{1}{10}$

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(iv) $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

$\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

$= \frac{(3+2i)(2+5i)+(3-2i)(2-5i)}{(2-5i)(2+5i)}$

$= \frac{(6+15i+4i+10i^2)+(6-15i-4i+10i^2)}{(2)^2-(5i)^2}$

$= \frac{6+19i-10+6-19i-10}{4-25i^2}$ $[i^2 = -1]$

$= \frac{12-20}{4+25} = \frac{-8}{29} = \frac{-8}{29} + 0\text{i}$ $[i^2 = -1]$

$\therefore a = \frac{-8}{29}$ and b = 0

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(v) $\frac{2+\sqrt{-3}}{4+\sqrt{-3}}$

$$\frac{2+\sqrt{-3}}{4+\sqrt{-3}}$$

$$= \frac{2+\sqrt{3\times(-1)}}{4+\sqrt{3}\times(-1)}$$

$$= \frac{2+\sqrt{3\sqrt{-1}}}{4+\sqrt{3\sqrt{-1}}}$$

$$= \frac{2+\sqrt{3i}}{4+\sqrt{3i}}$$

$$= \frac{(2+\sqrt{3i})\ (4-c)}{(4+\sqrt{3i})((4-\sqrt{3i})}$$ {Multiply and divide by (4 - $\sqrt{3i}$)}

$$= \frac{8-2\sqrt{3i}+4\sqrt{3i}-3i^2}{(4)^2-(\sqrt{3i})^2}$$

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$$= \frac{8+2\sqrt{3i}+3}{16-3i}$$

$$= \frac{11+2\sqrt{3i}}{16+3}$$ $\{since\ ,i^2\ =-1\}$

$$= \frac{11+2\sqrt{3i}}{19} = \frac{11}{19} + \frac{2\sqrt{3}}{19}i$$

$$= \text{a} = \frac{11}{19} \text{ and b} = \frac{2\sqrt{3}}{19}$$

vi) (2+3i)(2−3i)

(2+3i)(2−3i)

$= \quad (4)^2 - (3i)^2$

$= \quad 4 - 9i^2$

$= \quad 4 + 9 \qquad \{since\ ,\ i^2 = -1\}$

$= \quad 13$

$= \quad 13 + 0i$

$\therefore a = 13\ and\ b = 0$

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(vii) $\frac{4i^8-3i^9+3}{3i^{11}-4i^{10}-2}$

Solution:

$$\frac{4i^8-3i^9+3}{3i^{11}-4i^{10}-2}$$

$$=\frac{4(i^4)^2-3i^8i+3}{3i^8i^3-4i^8i^2-2}$$

$$=\frac{4(i^4)^2-3(i^4)i+3}{3(i^4)^2i^3-4(i^4)^2i^2-2}$$

$$=\frac{4(1)^2-3(1)^2i+3}{3(1)^2(-i)-4(1)^2(-1)-2}$$ $\{since\ ,\ i^2=-1\ ,i^3=-i,i^4=1\}$

$$=\frac{4-3i+3}{-3i+4-2}$$

$$=\frac{7-3i}{2-3i}$$

$$=\frac{(7-3i)(2+3i)}{(2-3i)(2+3i)}$$ {Multiply and divide by $(2+3i)$}

$$=\frac{14+21i-6i-9i^2}{(2)^2\ (3i)^2}$$

$$=\frac{14+15i+9i}{4+9}$$

$$=\frac{23+15i}{13}=\frac{23}{13}+\frac{15}{13}i$$

$\therefore$ a $=\frac{23}{13}$ and b $=\frac{15}{13}$

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(3) $(-1+\sqrt{3}i)^3$

$= (-1)^3 + 3(-1)^3(\sqrt{3}i)+3(-1)(\sqrt{3}i)^2 + \sqrt{3}i^3$ $\{since\ ,\ (a+b)^3 = (a)^3 + 3a^2 b + 3ab^2 + b^3\}$

$= -1+3\sqrt{3}i-3(3i^2)+(3\sqrt{3}i^3)$

$= -1+3\sqrt{3}i + 9 - 3\sqrt{3}i^3)$

$= \left(-1 + 3\sqrt{3}i)^3\right)$

$= 8$, which is a real number

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(4) Evaluate the following :

i) i^{35}

whenever the index is a bigger number we need to expressit in the form of 4

i.e.35 = 32+3=(4× 8) + 3

$\therefore\ i^{35} = i^{32} i^{3} = (i^{4})^{8} = (i)^{3}\ \{since, i^{3}\ = -i\ and\ i^{4}\ = 1\}$
$= \ (1)^{8}(-i) = -i$

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ii)) i^{888}

Since ,888 = 4 × 222

∴ i^{888}=$(i^4)^{222} = (1)^{222}$=1 $\{since, i^4 = 1\}$

iii)i^{93}

i^{93}

Since ,93 = (4 × 23)+1

∴ $i^{93} = c^{23}\, i = (1)^{23} i = i\ \{since, i^4 = 1\}$

(iv) i^{116}

Since ,116 = 4 × 29

∴ $i^{116} = (i^4)^{29} = (1)^{29}$ =1 $\{since, i^4 = 1\}$

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v) i^{403}

i^{403}

Since ,$403 = (4 \times 100)+3$

$\therefore\ i^{403} = (i^4)^{100}(i^3)= (1)^{100}\ (-i) = -i$ {since $i^3 = -\text{i}, i^4 = 1$}

vi) $\frac{1}{i^{58}}$

$\frac{1}{i^{58}}$

Since ,$58 = (4 \times 14)+2$

$\therefore\ i^{58} = (i^4)^{14}(i^2)= (1)^{14}\ (-1) = -1$ {since $i^2 = -1$ and $i^4 = 1$}

$\therefore \frac{1}{i^{58}} = \frac{1}{-1} = -1$

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vii) $i^{30} + i^{40} + i^{50} + i^{60}$

Solution:

Since ,$30 = 4 \times 7+2$, $40 = 4 \times 10$,

$50 = 4 \times 12+2$, $60 = 4 \times 15$

$\therefore i^{30} = i^{28}\, i^2 = (i^4)^7\, i^2$

$= (1)^7(-1) = -1$

$i^{40} = (i^4)^{10}$

$= (1)^{10} = 1$

$i^{50} = i^{48}\, i^2 = (i^4)^{12}\, i^2$

$= (1)^{12}(-1) = -1$ {since $i^2 = -i$and $i^4 = 1$}

$i^{60} = (i^4)^{15}$

$= (1)^{15} = 1$

$\therefore i^{30} + i^{40} + i^{50} + i^{60} = -1 + 1 - 1 + 1 = 0$

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5) Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number

$$i^{10} = i^8\ i^2 = (i^4)^2\ i^2$$
$$= (1)^2\ \ (-1) = -1$$

$$i^{20} = (i^4)^5$$

$$= (1)^5 \quad = 1$$

$$i^{30} = i^{28}\, i^2 = (i^4)^7\ i^2$$

$$= (1)^7\ \ (-1) = -1\ \ \{\text{since } i^2\ = -i \text{and } i^4\ = 1\}$$

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$\therefore\ 1 + i^{10} + i^{20} + i^{30} = 1 - 1 + 1 - 1 = 0,$ which is a real number

6) Find the value of

$i)\ i^{49}+i^{68}+i^{89}+i^{110}$

$i^{49}+i^{68}+i^{89}+i^{110}$

$\therefore\ i^{49}=i^{48}\,i=\ (i^4)^{12}\,i=(1)^{12}\,i=i$

$i^{68}\ =(i^4)^{17}=(1)^{17}=1$

$i^{89}\ =i^{88}\,i=\ (i^4)^{22}\,i=(1)^{22}\,i=i$

$i^{110}\ =i^{108}\ i^2=\ (i^4)^{27}i^2\ =(1)^{27}(-1)=-1$

$i^{49}+i^{68}+i^{89}+i^{110}\ =i+1+i-1=2i$

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6) ii) $i + i^2 + i^3 + i^4$

$i + i^2 + i^3 + i^4 = i - 1 - i + 1 = 0$

7) Find the value of $1 + i^2 + i^4 + i^6 + i^8 + \cdots \ldots + i^{20}$

$1 + i^2 + i^4 + i^6 + i^8 + \cdots \ldots + i^{20}$

Since , $i^2, i^6, i^{10}, i^{14}, i^{18} = -1$

$i^4, i^8, i^{12}, i^{16}, i^{18} = 1$

$\therefore 1 + (i^2 - i^4) + (i^6 + i^8) + \cdots . + (i^{18} + i^{20})$

$= 1 + 0 + 0 + \cdots + 0$
$= 1$

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8) Find the value of x and y which satisfy the following equations ($x, y \in R$)

i) $(x+2y) + (2x-3y)\, i + 4i = 5$

$(x+2y) + (2x-3y)\, i + 4i = 5$

$\therefore (x+2y) + (2x-3y)\, i = 5 - 4i$

Comparing we get ,

$x + 2y = 5$ ....(i)

$2x - 3y = -4$ .... (ii)

Multiply equation (i) by 2 and subtract it from (ii)

$\therefore 2x - 3y = -4$

$2x + 4 = 10$

$(-) \quad (-) \quad (-)$

$-7y = -14$

$\therefore \quad y = 2$

substitute y=2 in equation (i)

$x + 2(2) = 5$

$\therefore \quad x + 4 = 5 \therefore x = 1$

$\therefore$ x=1 and y=2

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ii) $\frac{x+1}{1+i} + \frac{y-1}{1-i} = i$

$$\frac{x+1}{1+i} + \frac{y-1}{1-i} = i$$

$$\therefore \frac{(x+1)(1-i)+(y-1)(91+i)}{(1-i)(1-i)} = i$$

$$\frac{x+1-xi-i+y+yi-1-i}{1^2+i^2} = i$$

$$\frac{x+xi-i+y+yi-1-i}{1^2+i^2} = i$$

$$\frac{x+xi-i+y+yi-i}{1+1} = i$$

$$(x+y) + (-xi+yi) - 2i = 2i$$

$$(x+y) + (-x+y)\,i = 4i$$

Comparing we get ,

$$x + y = 0$$

$$\therefore \quad -x + y = 4$$

Adding equations (i) and (ii),we get

$$x + y = 0$$

$$-x + y = 4$$

$$2y = 4$$

$$y = 2$$

∴ Substitute y = 2 in equation (i)

$$x + 2 = 0$$

$$\therefore x = -2$$

$$\therefore x = -2 \text{ and } y = 2$$

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9) Find the value of

(i) $x^3 - x^2 + x + 46$, if $x = 2 + 3i$

$x^3 - x^2 + x + 46$

since , $x = 2 + 3i$

$\therefore x - 2 = 3i$

$(x - 2)^2 = 3i^2$

$\therefore\ x^2 - 4x + 4 = 9i^2$

$\therefore\ x^2 - 4x + 4 = -9$

$\therefore\ x^2 - 4x + 13 = 0$

divide equation (i) by equation (ii)

$$
\begin{array}{r|l}
 & x + 3 \\
x^2 - 4x + 13 & x^3 - x^2 + x + 46 \\
 & x^3 - 4x^2 + 13x \\
 & (-)\quad (+)\qquad (-) \\
\hline
 & 3x^2 - 12x + 46 \\
 & 3x^2 - 12x + 39 \\
 & (-)\quad (+)\quad (-) \\
\hline
 & 7
\end{array}
$$

$\therefore$

$\therefore\ x^3 - x^2 + x + 46 = (x + 3)(x^2 - 4x + 13) + 7$

$= (x + 3)(0) + 7$ {from (ii)}

$= 0 + 7 = 7$

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ii) $2x^3 - 11x^2 + 44x + 27, if\ x = \frac{25}{3-4i}$

Sol:

$2x^3 - 11x^2 + 44x + 27$ ...... (i)

$x = \frac{25}{3-4i} \times \frac{3+4i}{3+4i}$

$= \frac{25(3+4i)}{(3)^2-(4i)^2}$

$= \frac{25(3+4i)}{9-16i^2}$

$= \frac{25(3+4i)}{25}$

$x = 3 + 4i$

$\therefore x - 3 = 4i$

$(x-3)^2 = (4i)^2$ {squaring both sides }

$\therefore x^2 - 6x + 9 = -16i^2$

$\therefore x^2 - 6x + 25 = 0$ . ... (ii)

Divide (i) by (ii)

$$
\begin{array}{r|l}
 & 2x + 1 \\
x^2 - 6x + 25 & 2x^3 - 11x^2 + 44x + 27 \\
 & 2x^3 - 12x^2 + 50x \\
 & (-) \quad (+) \quad (-) \\
\hline
 & x^2 - 6x + 27 \\
 & x^2 - 6x + 25 \\
 & (-) \quad (+) \quad (-) \\
\hline
 & 2
\end{array}
$$

$\therefore 2x^3 - 11x^2 + 44x + 27 = x^2 - 6x + 25\ (2x+1)+2$

$= 0(2x+1) + 2$ $\{from(ii)\}$

$= 0 + 2$

$= 2$

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