CHAPTER 1 : SETS

EXERCISE 1.1

1. Describe the following sets in Roster form

i){x/x is a letter of the word 'MARRIAGE'} ii) $\{x \mid x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\}$

iii)$\{x \mid x = 2n, n \in N\}$

Solution :

i. Let A ={x/x is a letter of the word 'MARRIAGE'}

$\therefore \quad A = \{M, A, R, I, G, E\}$

II.) Let $B = \{x \mid x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\}$

$\therefore \quad B = \{0,1,2,3,4\}$

iii.) Let $C = \{x \mid x = 2n, n \in N\}$

$\therefore \quad C = \{2,4,6,8, \ldots\}$

2. Describe the following sets in Set–Builder form

i) {0} ii) {0, ±1, ±2, ±3} iii) $\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\}$

Solution :

i.) Let A = {0}

0 is a whole no. but it is not a natural no.

$\therefore \quad A = \{x | \, x \in W, x \notin N\}$

ii.) Let B = {0, ±1, ±2, ±3}

B is the set of elements which belongs to integers(I) from -3 to 3.

$\therefore \quad B = \{x | \, x \in i, -3 \leq x \leq 3\}$

iii.) Let C = $\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\}$

$\therefore \quad C = \{x | \, x = \frac{n}{n^2+1}, n \in N, n \leq 7\}$

3. IF A={x | $6x^2 + x - 15 = 0$} B={$x \mid 2x^2 - 5x - 3 = 0$} C={x | $2x^2 - x - 3 = 0$}

Find i) (A∪B∪C) ii) (A∩B∩C)

Solution:

A={x | $6x^2 + x - 15 = 0$}

$\therefore \quad 6x^2 + x - 15 = 0$

$\therefore \quad 6x^2 + 10x - 9x - 15 = 0$

$\therefore \quad 2x(3x + 5) - 3(3x + 5) = 0$

$\therefore \quad (3x + 5)(2x - 3) = 0$

$\therefore \quad x = -\frac{5}{3} \; or \; x = \frac{3}{2}$

$\therefore \quad A = \left\{-\frac{5}{3}, \frac{3}{2}\right\}$

$\therefore$ B={$x \mid 2x^2 - 5x - 3 = 0$}

$\therefore \quad 2x^2 - 5x - 3 = 0$

$\therefore \quad 2x^2 - 6x + x - 3 = 0$

$\therefore \quad 2x\,(x - 3) + 1(x - 3) = 0$

$\therefore \quad (x - 3)(2x + 1) = 0$

$\therefore \quad x - 3 = 0 \quad or \quad 2x + 1 = 0$

$\therefore \quad x = 3 \quad or \quad x = -\frac{1}{2}$

$\therefore \quad B = \left\{3. -\frac{1}{2}\right\}$

C={$x \mid 2x^2 - x - 3 = 0$}

$\therefore \quad 2x^2 - x - 3 = 0$

$\therefore \quad 2x^2 - 3x + 2x - 3 = 0$

$\therefore \quad x(2x - 3) + 1(2x - 3) = 0$

$\therefore \quad (2x - 3)(x + 1) = 0$

$\therefore \quad 2x - 3 = 0 \quad or \quad x + 1 = 0$

$\therefore \quad x = \frac{3}{2} \quad or \quad x = -1$

$\therefore$ C = $\left\{\frac{3}{2}, -1\right\}$

i) (A∪B∪C) = $\left\{-\frac{5}{3}, \frac{3}{2}\right\} \cup \left\{3, -\frac{1}{2}\right\} \cup \left\{\frac{3}{2}, -1\right\}$

= $\left\{-\frac{5}{3}, -1, -\frac{1}{2}, \frac{3}{2}. 3\right\}$

ii) (A∩B∩C)={ }

4) If A, B, C are the sets for the letters in the words 'college', 'marriage' and 'luggage‘ respectively, then verify that

[A-(B∪C) = (A-B)∩(A-C)].

Solution:

$A = \{c, o, l, g, e\}$

$B = \{m, a, r, i, g, e\}$

$C = \{l, u, g, a, e\}$

$B \cup C = \{m, a, r, i, g, e, l, u\}$

$\therefore \quad A - (B \cup C) = \{c, o\}$ .....(i)

$A - B = \{c, o, l\}$

$A - C = \{c, o\}$

$\therefore$ [(A-B) ∩ (A-C)] = $\{c, o\}$.....(ii)

From i & ii we can say that

$[A - (B \cup C)]$ = [(A-B) ∩ (A-C)]

5) If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

i) A∪(B∩C) = (A∪B) ∩(A∪C)

Solution:

$A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$C = \{4,5,6,7,8\}$

$X = \{1,2,3,4,5,6,7,8,9,10\}$

i.) $B \cup C = \{4,5,6,\}$

∴ A ∪ (B ∩ C) = {1,2,3,4,5,6} ….(i)

$A \cup B = \{1,2,3,4,5,6\}$

$A \cup C = \{1,2,3,4,5,6,7,8,\}$

∴ (A ∪ B) ∩ (A ∪ C)={1,2,3,4,5,6} ….(ii)

From (i) and (ii), we get

A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

5) If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

ii) A∩(B∪C) = (A∩B) ∪(A∩C)

Solution:

$A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$C = \{4,5,6,7,8\}$

$X = \{1,2,3,4,5,6,7,8,9,10\}$

ii.) $B \cup C = \{3,4,5,6,7,8,\}$

∴ A ∩ (B ∪ C)={3,4} …(i)

$A \cap B = \{3,4,\}$

$A \cap C = \{4\}$

∴ (A ∩ B) ∪ (A ∩ C) ={3,4} …(ii)

From (i) and (ii), we get

A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

5) If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

iii) (A∪B)' = (A'∩B')

Solution:

$A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$C = \{4,5,6,7,8\}$

$X = \{1,2,3,4,5,6,7,8,9,10\}$

iii.) $A \cup B = \{1,2,3,4,5,6\}$

$\therefore$ (A∩B)‘ $= \{7,8,9,10\}$ …(i)

$A' = \{5,6,7,8,9,10\}$,

$B' = \{1,2,7,8,9,10\}$

$\therefore$ $A' \cap B' = \{7,8,9,10\}$ …(ii)

From (i) and (ii), we get

(A∪B)' = (A'∩B')

5) If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

iv) (A∩ B)' = A'∪ B '

Solution:

$A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$C = \{4,5,6,7,8\}$

$X = \{1,2,3,4,5,6,7,8,9,10\}$

iv.) $A \cap B = \{3,4\}$

$\therefore (A \cap B)' = \{1,2,5,6,7,8,9,10\}$ …(i)

$A' = \{5,6,7,8,9,10\}$

$B' = \{1,2,7,8,9,10\}$

$\therefore A' \cap B' = \{1,2,5,6,7,8,9,10\}$ …(ii)

From (i) and (ii), we get

(A∩B)' = A'∪B'

5) If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
then verify the following:
v) A = (A∩B)∪ (A∩B′)

Solution:

$A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$C = \{4,5,6,7,8\}$

$X = \{1,2,3,4,5,6,7,8,9,10\}$

v.) $A = \{1,2,3,4\}$ …(i)

$A \cap B = \{3,4\}$

$B' = \{1,2,7,8,9,10\}$

$A \cap B' = \{1,2\}$

∴ (A∩B)∪ (A∩B′)={1,2,3,4} ….(ii)

From (i) and (ii), we get

A = (A∩ B)∪ (A∩ B′)

5) If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

vi) B = (A∩B)∪ (A'∩B)

Solution:

$A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$C = \{4,5,6,7,8\}$

$X = \{1,2,3,4,5,6,7,8,9,10\}$

vi.) $B = \{3,4,5,6\}$ …(i)

$A \cap B = \{3,4\}$

$A' = \{5,6,7,8,9,10\}$

$A' \cap B = \{5,6\}$

∴ $(A \cap B) \cup (A' \cap B) = \{3,4,5,6\}$ …(ii)

From (i) and (ii), we get

B = (A∩ B)∪ (A'∩ B)

5) If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

vii) n (A∪B) = n(A) + n(B) - n(A∩B)

Solution:

$A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$

$C = \{4,5,6,7,8\}$

$X = \{1,2,3,4,5,6,7,8,9,10\}$

vii.) $A = \{1,2,3,4\}$ $B = \{3,4,5,6\}$,

$A \cap B = \{3,4\}$, $A \cup B = \{1,2,3,4,5,6\}$

$\therefore$ $n(A) = 4$, $n(B) = 4$,

$n(A \cap B) = 2$,

$n(A \cup B) = 6$ …(i)

$\therefore$ $n(A) + n(B) - n(A \cap B) = 4 + 4 - 2$

$\therefore$ $n(A) + n(B) - n(A \cap B) = 6$ …(ii)

From (i) and (ii), we get

n (A∪B) = n(A) + n(B) - n(A∩B)

Properties of Cardinality of Sets :

For 2 Sets :

i. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii. $n(A \cap B') + n(A \cap B) = n(A)$

iii. $n(A' \cap B) + n(A \cap B) = n(B)$

iv. $n(A \cap B') + n(A \cap B) + n(A' \cap B) = n(A \cup B)$

For 3 Sets :

$i.e\ (A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

6) If A and B are subsets of the universal set X and $n(X) = 50$, $n(A) = 35$, $n(B) = 20$, $n(A' \cap B') = 5$,

find:- i) $n(A \cup B)$ ii) $n(A \cap B)$ iii) $n(A' \cap B)$ iv) $n(A \cap B)'$

Solution:

$$n(X) = 50,\ n(A) = 35, n(B) = 20, n(A' \cap B') = 5$$

i.) $n(A \cup B) = n(X) - [n(A \cup B)']$

$= n(X) - n(A' \cap B')$

$= 50 - 5$

$= 45.$

ii.) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 35 + 20 - 45$

$= 10$

iii.) $n(A' \cap B) = n(B) - n(A \cap B)$

$= 20 - 10$

$= 10.$

iv.) $n(A \cap B') = n(A) - n(A \cap B)$

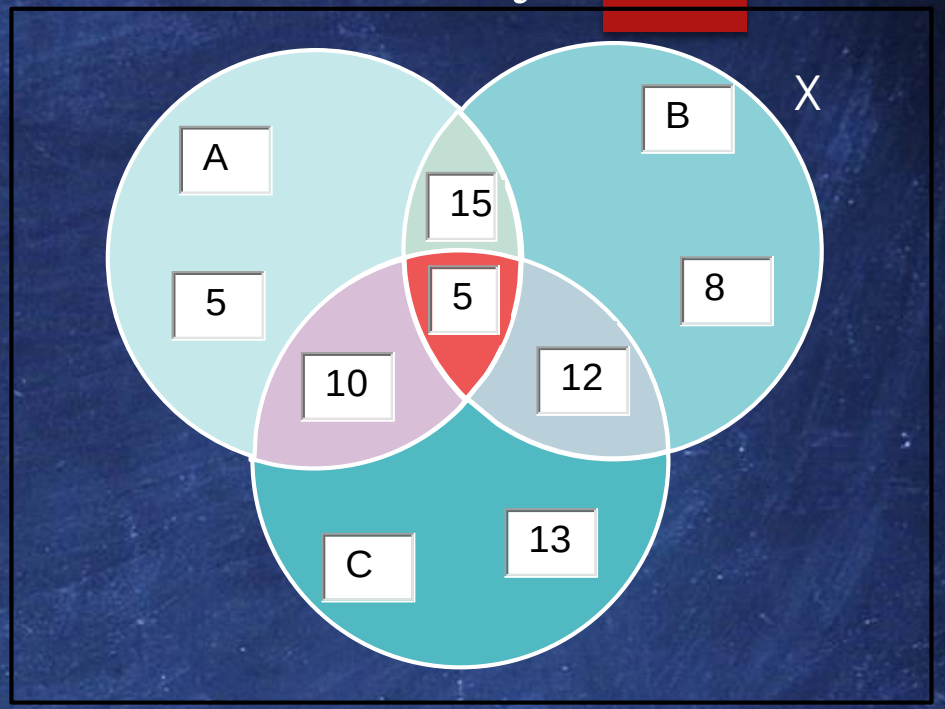
$= 35 - 10$

$= 25$

7) Out of 200 students; 35 students failed in MHT-CET, 40 in AIEEE and 40 in IIT entrance, 20 failed in MHT-CET and AIEEE, 17 in AIEEE and IIT entrance, 15 in MHT-CET and IIT entrance and 5 failed in all three examinations. Find how many students.

i) did not fail in any examination.

ii) failed in AIEEE or IIT entrance.



Solution:

LET A = set of students who failed in MHT-CET

B = set of students who failed in AIEEE

C = set of students who failed in IIT entrance

X = set of all students.

$\therefore$ $n(X) = 200, n(A) = 35, n(B) = 40, n(C) = 40,$
$n(A \cap B) = 20, n(B \cap C) = 17, n(A \cap C) = 15,$
$n(A \cap B \cap C) = 5$

i.) $n(A \cup B \cup C)$
$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) -$
$n(A \cap C) + n(A \cap B \cap C).$
$= 35 + 40 + 40 - 20 - 17 - 15 + 5$
$= 68$

$\therefore$ $No. of\ students\ who\ did\ not\ fail\ in\ any\ exam$
$= n(X) - n(A \cup B \cup C)$
$= 200 - 68$
$= 132$

ii) $No\ of\ students\ who\ failed\ in\ AIEEE\ or\ IIT\ entrance = n(B \cup C)$
$= n(B) + n(C) - n(B \cap C)$
$= 40 + 40 - 17$
$= 80 - 17$
$= 63.$

8) From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read.

i) at least one of the newspapers.

ii) neither Marathi nor English newspaper.

iii) Only one of the newspapers.

Solution:

LET $M = set\ of\ individuals\ who\ read\ marathi\ newspapers.$

$E = set\ of\ individuals\ who\ read\ English\ newspapers.$

$X = set\ of\ all\ literate\ indiduals.$

$\therefore \quad n(X) = 2000,$

$n(M) = \frac{70}{100} \times 2000 = 1400$

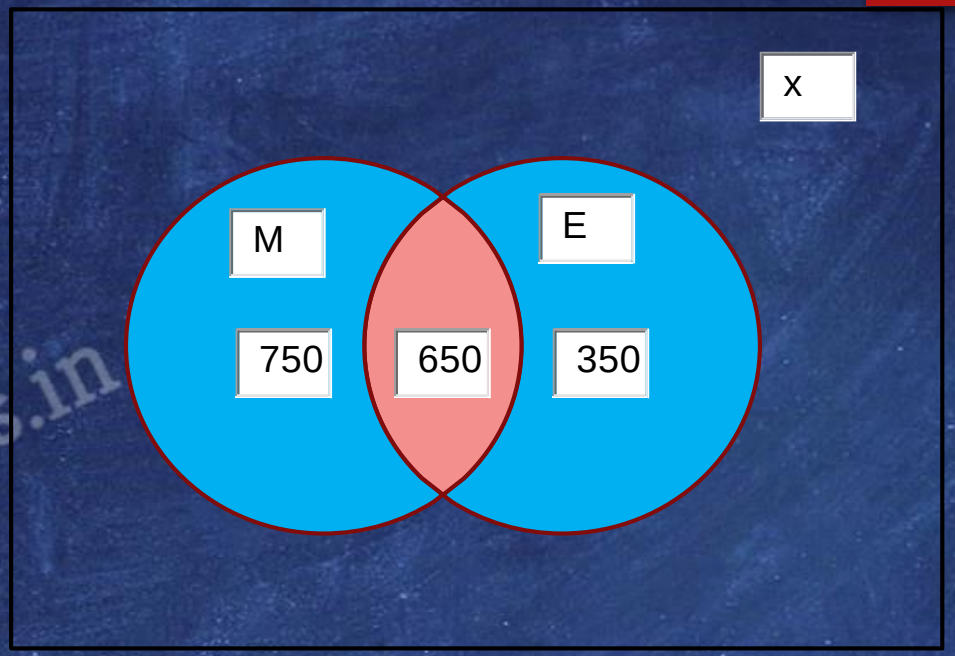
$n(E) = \frac{50}{100} \times 2000 = 1000$

$n(M \cap E) = \frac{32.5}{100} \times 2000 = 650$

$n(M \cup E) = n(M) + n(E) - n(M \cap E)$

$= 1400 + 1000 - 650$

$= 1750$



i.) No. of individuals who read at least one of the newspaper = $n(M \cup E)$ =1750.

ii.) No. of individuals who read neither Marathi nor English newspaper = $n(X) - N(M \cup E)$

$= 2000 - 1750$

$= 250.$

iii.) No. of individuals who read Only one of the newspaper = $n(M \cup E) - n(M \cap E)$

$= 1750 - 650$

$= 1100.$

9) In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 student take both tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone takes atleast one beverage, find the number ofstudents in the hostel.

Solution:

LET $T = set\ of\ students\ who\ take\ tea$

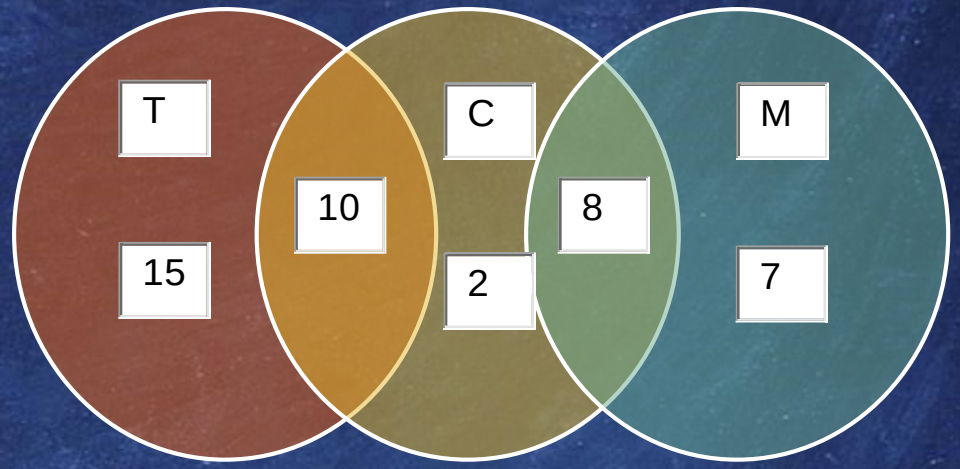
$C = set\ of\ students\ who\ take\ coffee$

$M = set\ of\ students\ who\ take\ milk$

$\therefore \quad n(T) = 25,\ n(C) = 20,\ n(M) = 15,$

$n(T \cap C) = 10, n(M \cap C) = 8, n(T \cap M) = 0$

$n(T \cap M \cap C) = 0$



$\therefore$ Number of students in the hostel

$= n(T \cup C \cup M)$

$= n(T) + n(c) + n(M) - n(T \cap C) - n(M \cap C) - (T \cap M) + n(T \cup M \cup C)$

$= 25 + 20 + 15 - 10 - 8 - 0 + 0$

$= 42$

10) There are 260 persons with a skin disorder. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to

i) Chemical A but not Chemical B ii) Chemical B but not Chemical A

iii) Chemical A or Chemicalpersons expose

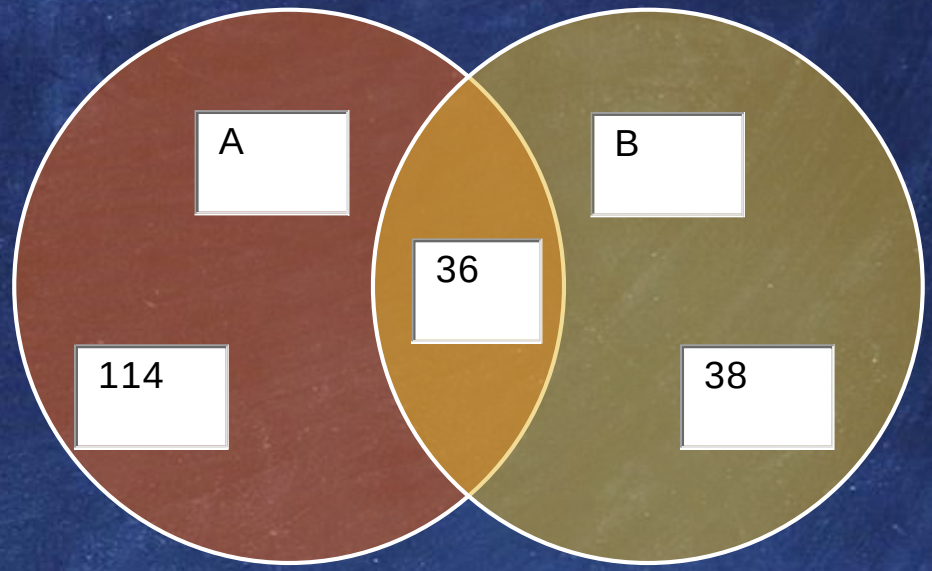
Solution:

LET $A = set\ of\ persons\ who\ exposed\ to\ chemical\ A$

$B = set\ of\ persons\ who\ exposed\ to\ chemical\ B$

$X = set\ of\ all\ persons$

$\therefore \quad n(X) = 260, n(A) = 150, n(B) = 74, n(A \cap B) = 36$



i.) $No.of\ who\ exposed\ to\ chemical\ A\ but\ not\ to\ chemical\ B$

$= n(A \cap B')$

$= n(A) - n(A \cap B)$

$= 150 - 36$

$= 114$

ii.) $No.of\ who\ exposed\ to\ chemical\ B\ but\ not\ to\ chemical\ A$

$= n(A' \cap B)$

$= n(B) - n(A \cap B)$

$= 74 - 36$

$= 38$

iii.) $No.of\ persons\ exposed\ to\ chemical\ A\ or\ chemical\ B$

$= n(A \cup B)$

$= n(A) + n(B) - n(A \cap B)$

$= 150 + 74 - 36$

$= 188$

11) If A = {1, 2, 3}, write the of all possible subset of A.

Solution:

$A = \{1,2,3\}$

∴ { }, { 1 } , { 2 } , { 3 } , { 1,2 } , { 2,3 } , { 1,3 } and { 1,2,3 } are all the possible subsets of A.

12) Write the following intervals in set-builder form.

i) (-3, 0)

ii) [6,12]

iii) (6, 12)

iv) (-23, 5)

Solution:

i.) $(-3,0)$

$= \{x | x \in R, -3 < x < 0\}$

ii.) $[6,12]$

$= \{x | x \in R, 6 \leq x \leq 12\}$

iii.) $(6,12)$

$= \{x | x \in R, 6 < x < 12\}$

iv,.) $(-23,5)$

$= \{x | x \in R, -23 < x < 5\}$