EXERCISE 1.2

1) If $(x - 1, y + 4) = (1, 2)$ find the values of x and y.

Solution:

$$(x-1, y+4) = (1,2)$$

$By\ the\ defination\ of\ equality\ of\ ordered\ pairs\ we\ have,$

$$x - 1 = 1 \quad and \quad y + 4 = 2$$

$$\therefore \quad x = 2 \quad and \quad y = -2$$

2) If $\left(x+\frac{1}{3}, \frac{y}{3}-1\right) = \left(\frac{1}{3}, \frac{3}{2}\right)$, $find\ x\ and\ y.$

Solution:

$$\left(x+\frac{1}{3}, \frac{y}{3}, -1\right) = \left(\frac{1}{3}, \frac{3}{2}\right)$$

$By\ the\ defination\ of\ equality\ of\ ordered\ pairs\ we\ have,$

$$x + \frac{1}{3} = \frac{1}{3} \quad and \quad \frac{y}{3} - 1 = \frac{3}{2}$$

$$\therefore \quad x = \frac{1}{3} - \frac{1}{3} \quad and \quad \frac{y}{3} = \frac{3}{2} + 1 = \frac{5}{2}$$

$$\therefore \quad \mathrm{x} = 0 \ \text{ and } \ \mathrm{y} = \frac{15}{2}$$

5) Let A = { 1, 2, 3, 4}, B = { 4, 5, 6}, C = { 5, 6}.

Find i) A × (B ∩ C) ii) (A × B) ∩ (A × C) iii) A × (B ∪ C) vi) (A × B) ∪ (A × C)

Solution :

i) A × (B ∩ C)

$\because B \cap C = \{5,6\}$

$\therefore A \times (B \cap C) = \{(1,5),(1,6),(2,5),(2,6),(3,5),(3,6),(4,5),(4,6)\}$

Solution :

ii) (A × B) ∩ (A × C)

$(A \times B) = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(4,4),(4,5),(4,6)\}$

$(A \times C) = \{(1,5),(1,6),(2,5),(2,6),(3,5),(3,6),(4,5),(4,6)\}$

$\therefore (A \times B) \cap (A \times C) = \{(1,5),(1,6),(2,5),(2,6),(3,5),(3,6),(4,5),(4,6)\}$

Solution :

iii) A × (B ∪ C)

$\because (B \cup C) = \{4,5,6\}$

$\therefore A \times (B \cup C) = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(4,4),(4,5),(4,6)\}$

Solution :

vi) (A × B) ∪ (A × C)

$A \times B = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4)(3,5),(3,6),(4,4),(4,5),(4,6)\}$

$A \times C = \{(1,5),(1,6),(2,5),(2,6),(3,5),(3,6),(4,5),(4,6)\}$

$\therefore (A \times B) \cup (A \times C) = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(4,4),(4,5)(4,6)\}$

6) Express {(x, y) / $x^2 + y^2 = 100$ where $x, y \in$ W} as a set of ordered Pairs.

Solution :

{(x, y) / $x^2 + y^2 = 100$ where $x, y \in$ W}

We have, $x^2 + y^2 = 100$

When $x = 0$ *and* $y = 10$

$\therefore x^2 + y^2 = 0^2 + 10^2 = 0 + 100 = 100$

When $x = 6$ *and* $y = 8$

$\therefore x^2 + y^2 = 6^2 + 8^2 = 36 + 64 = 100$

When $x = 8$ *and* $y = 6$

$\therefore x^2 + y^2 = 8^2 + 6^2 = 64 + 36 = 100$

When $x = 10$ *and* $y = 0$

$\therefore x^2 + y^2 = 10^2 + 0^2 = 100 + 0 = 100$

$\therefore$ *Set of order pairs* $= \{(0,10), (6,8), (8,6), (10,0)\}$

7) Write the domain and range of the following relations.

i) {(a, b) / a ∈ N, a<6 and b = 4} ii) {(a, b) / a, b ∈ N, a+b = 12} iii){(2, 4), (2, 5), (2,6), (2, 7)}

Solution :

i) Let R_1 = {(a, b) / a ∈ N, a<6 and b = 4}

Set of values of 'a' are domain and set of value of 'b' are range.

$a \in N \; and \; a < 6$

$\therefore a = 1,2,3,4,5 \; and \; b = 4$

Domain $(R_1) = \{1,2,3,4,5\}$

Range $(R_1) = \{4\}$

ii) Let R_1 ={(a, b) / a, b ∈ N, a+b = 12}

Now, $a, b \in N \; and \; a + b = 12$

When $a = 1, b = 11$

When $a = 2, b = 10$

When $a = 3, b = 9$

When $a = 4, b = 8$

When $a = 5, b = 7$

When $a = 6, b = 6$

When $a = 7, b = 5$

When $a = 8, b = 4$

When $a = 9, b = 3$

When $a = 10, b = 2$

When $a = 11 \; b = 1$

$\therefore$ Domain $(R_2) = \{1,2,3,4,5,6,7,8,9,10,11\}$

Range $(R_2) = \{11,10,9,8,7.6,5,4,3,2,1\}$

iii. Let R_3 = {(2, 4), (2, 5), (2,6), (2, 7)}

Domain $R_3 = \{2\}$

Range $R_3 = \{4,5,6,7\}$

8) Let A = {6, 8} and B = {1, 3, 5} Let R = {(*a*, *b*)/ *a*∈ A, *b*∈B, *a*-*b* is an even number} Show that R is an empty relation from A to B

Solution :

$A = \{6,8\}, \qquad B = \{1,3,5\}$

$R = \{(a,\ b)/\ a \in$ A, $b \in$ B, $a-b$ is an even number}

$a \in A$

$\therefore\ a = 6,8$

$b \in B$

$\therefore\ b = 1,3,5$

$When\ a = 6\ and\ b = 1, a - b = 5\ which\ is\ odd$

$When\ a = 6\ and\ b = 3, a - b = 3\ which\ is\ odd$

$When\ a = 6\ and\ b = 5\ a - b = 1\ which\ is\ odd$

$When\ a = 8and\ b = 1, a - b = 7\ which\ is\ odd$

$When\ a = 8\ and\ b = 3, a - b = 5\ which\ is\ odd$

$When\ a = 8\ and\ b = 5, a - b = 3\ which\ is\ odd$

Thus, no set of value of a and b gives a-b even

∴ R is an empty relation from A to B.

9) Write the relation in the Roster form and hence find its domain and range.

i) $R_1 = \{(a, a^2) | a \text{ is a prime number less than } 15\}$

ii) $R_2 = \left\{\left(a, \frac{1}{a}\right) \middle| 0 < a \leq 5, a \in N\right\}$

Solution :

i. $R_1 = \{(a, a^2) | a \text{ is a prime number less than } 15\}$

$\therefore a = 2, 3, 5, 7, 11, 13$

$\therefore a^2 = 4, 9, 25, 49, 121, 169$

$\therefore R_1 = \{(2,4), (3,9), (5,25), (7,49), (11,121), (13,169)\}$

$\therefore$ Domain $= \{a | a \text{ is a prime number less than } 15\} = \{2, 3, 5, 7, 11, 13\}$

Range $R_1 = \{(a, a^2) | a \text{ is a prime number less than } 15\} = \{4, 9, 25, 49, 121, 169\}$

ii. $R_2 = \left\{\left(a, \frac{1}{a}\right) \middle| 0 < a \leq 5. a \in N\right\}$

$\therefore a = 1, 2, 3, 4, 5$

$\therefore \frac{1}{a} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

$\therefore R_2 = \left\{(1,1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right), \left(5, \frac{1}{5}\right)\right\}$

$\therefore Domain\ (R_2) = \{a | 0 < a \leq 5, a \in N\} = \{1, 2, 3, 4, 5\}$

$\therefore$ Range $(R_2) = \left\{\frac{1}{a} \middle| 0 < a \leq 5, a \in N\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

10) R = {(a, b) / b = a + 1, a∈z, 0 < a < 5}.Find the Range of R.

Solution :

$R = \{(a, b) | b = a + 1, a \in Z, 0 < a < 5\}$

$\therefore\ a = 1, 2, 3, 4$

$\therefore\ b = 2, 3, 4, 5$

∴Range(R)={2,3,4,5}

11) Find the following relation as sets of ordered pairs.

i) $\{(x, y) / y = 3x, x \in \{1,2,3\}, y \in \{3,6,9,12\}\}$

ii) $\{(x, y) / y > x + 1, x \in \{1, 2\}$ and $y \in \{2, 4, 6\}\}$

iii) $\{(x, y) / x + y = 3, x, y \in \{0, 1, 2, 3\}\}$

Solution :

$i. \quad \{(x,y)|y = 3x, x \in \{1,2,3\}, y \in \{3,6,9,12\}\}$

Here $y = 3x$

$When\ x = 1, y = 3(1) = 3$

$When\ x = 2, y = 3(2) = 6$

$When\ x = 3, y = 3(3) = 9$

$\therefore Ordered\ pairs\ are\ \{(1,3),(2,6),(3,9)\}$

$ii. \{(x,y)|y > x + 1, x \in \{1,2\}\ and\ y \in \{2,4,6\}\}$

Here, $y > x + 1$

$When\ x = 1\ and\ y = 2, 2 \ngtr 1 + 1$

$When\ x = 1\ and\ y = 4, 4 > 1 + 1$

$When\ x = 1\ and\ y = 6, 6 > 1 + 1$

$When\ x = 2\ and\ y = 2, 2 \ngtr 2 + 1$

$When\ x = 2\ and\ y = 4, 4 > 2 + 1$

$When\ x = 2\ and\ y = 6, 6 > 2 + 1$

$\therefore$Ordered pairs are $\{(1,4), (1,6), (2,4), (2,6)\}$

iii. $\{(x,y)|x + y = 3, x, y \in \{0,1,2,3\}\}$

$Here\ x + y = 3$

$When\ x = 0, y = 3$

$When\ x = 1, y = 2$

$When\ x = 2, y = 1$

$When\ x = 3, y = 0$

$\therefore$ Ordered Pairs are

$\{(0,3),(1,2),(2,1),(3,0)\}$