Exercise 6.1

1. A teacher wants to select the class monitor in a class of 30 boys and 20 girls. In how many ways can he select a student if the monitor can be a boy or a girl?

Solution:

There are 30 boys and 20 girls in a class.

The teacher wants to select a class monitor from these boys and girls.

A boy can be selected in 30 ways and a girl can be selected in 20 ways.

By using fundamental principle of addition, number of ways either a boy or a girl is selected as a class monitor = 30 + 20 = 50.

2. In question 1, in how many ways can the monitor be selected if the monitor must be a boy? What is the answer if the monitor must be a girl?

.Solution:

Since there are 30 boys in the class

∴ A boy monitor can he selected in 30 ways.

ii. Since there are 20 girls in the class

∴ A girl monitor can be selected in 20 way,.

3. A Signal is generated from 2 flags by putting one flag above the other. If 4 flags of different colours are available, how many different signals can he generated?

Solution:

A signal is generated from 2 flags and there are 4 flags of different colours available.

∴ 1^{st} flag can be any one of the available 4 flags.

∴ It can be selected in 4 ways.

Now, 2^{nd} flag is to be selected for which 3 flags are available for a different signal.

∴ 2^{nd} flag can be anyone from these 3 flags.

∴ It can be selected in 3 ways.

∴ By using fundamental principle of multiplication,

Total number of ways in which a signal can be generated = 4 x 3 = 12

∴ 12 different signals can be generated.

4. How many two letter words can be formed using letters from the word SPACE, when
i. repetition of letters is allowed
ii. is not allowed
Solution:
Two-letter word is to be formed out of the letters of the word SPACE.

i. When repetition of the letters is allowed
1^{st} letter can be selected in 5 ways
2^{nd} letter can be selected in 5 ways
∴ By using fundamental principle of multiplication, total number of 2-letter words
= 5 x 5 = 25

ii. When repetition of the letters is not allowed
1^{st} letter can be selected in 5 ways
2^{nd} letter can be selected in 4 ways
∴ By using fundamental principle of multiplication, total number of 2-letter words
= 5 x 4 = 20

5. How many three-digit numbers can be formed from the digits 0, 1, 3, 5, 6 if
i. repetitions of digits are allowed
ii. are not allowed
Solution:
i. repetitions of digits are allowed
Three digit number is to be formed from the digits 0,1,3,5,6
When repetition of digits is allowed.
100's place digit should be a non zero number.
Hence, it can be any one from digits 1,3,5,6
∴ 100's place digit can be selected in 4 ways.
0 can appear in 10’s and unit’s place and digit can be repeated.
∴ 10's place digit can be selected in 5 ways and unit place digit can be selected in 5 ways
∴ By using - fundamental principle of multiplication, total number of three-digit numbers
= 4 x 5 x 5 = 100

ii. When repetition of digits is not allowed:
100's place digit should be a non zero number.
Hence, it can be any one from digits 1, 3, .5, 6
∴ 100's place digit can appear in 4 ways and unit's place digit can be selected in 3 ways
∴ By using fundamental principle of multiplication, total number of three-digit numbers
= 4 x 4 x 3 = 48

6. How many three-digit numbers can be formed using the digits 2, 3, 4, 5, 6 if digits can be repeated?

Solution:

A 3-digit number is to be formed from the digits 2, 3, 4, 5, 6 where digits can be repeated.

∴ Unit's place digit can be selected in 5 ways.

10's place digit can be selected in 5 ways.

100's place digit can be selected in 5 ways.

∴ By using fundamental principle of multiplication, total number of3-digit numbers

$= 5 \times 5 \times 5 = 125$

7. A letter lock has 3 rings and each ring has 5 letters. Determine the maximum number of trials that may be required to open the lock.

Solution:

A letter lock has 3 rings, each ring containing 5 different letters.

∴ A letter from each ring can be selected in 5 ways.

∴ By using fundamental principle of multiplication, total number of trials that can be made $=5 \times 5 \times 5 = 125$

Out of these 124 wrong attempts are made and in 125th attempt, the lock gets opened.

∴ Maximum number of trials required to open the lock are 125.

8. In a test that has 5 true/false questions, no student has got all correct answers and no sequence of answers is repeated. What is the maximum number of students for this to be possible?

Solution:

For a set of 5 true/false questions, each question can be answered in 2 ways.

∴ By using fundamental principle of multiplication, total number of possible sequences of answers $= 2 \times 2 \times 2 \times 2 \times 2 = 32$

Since, no student has written all correct answers

∴ Total number of sequences of answers given by the students in the class $= 32 - 1 = 31$

Also, no student has given the same sequence of answers.

∴ Maximum number of students in the class = Number of sequences of answers given by the students $= 31$

9. How many numbers between 100 and 1000 have 4 in the units place?

Solution:

Numbers between 100 and 1000 are 3-digit numbers.

A 3-digit number is to he formed from the digits 0, 1, 2, 3, 4, 5. 6, 7, 8, 9 where unit place digit is 4.

∵ Unit's place digit is 4.

∴ it can .be selected in 1 way only.

10's/place digit can he selected in 10 ways.

For 3-digit number 100's place digit should be a non-zero number.

∴ 100's place digit can be selected in 9 ways.

∴ By using fundamental principle of multiplication, total number of numbers between 100 and 1000 which have 4 in the units place = 1 x 10 x 9 = 90

10. How many numbers between 100 and 1000 have the digit 7 exactly once?

Solution:

Numbers between 100 and 1000 are 3-digit numbers.

A 3-digit number is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where exactly One of the digits is 7.

When 7 is in unit's place

Unit's place digit is 7.

∴ it can be selected in 1 way only.

10's place digit can be selected in 9 ways.

100's place digit can be selected in 8 ways.

∴ total number of numbers which have 7 in unit's place = 1 x 9 x 8= 72

When 7 is in 10's place

Unit's place digit can be selected in 9 ways.

10's place digit is 7

∴ it can be selected in 1 way only.

100's place digit can be selected in 8 ways.

∴ total number of numbers which have 7 in 10's place = 9 x 1 x 8 = 72

When 7 is in 100's place

Unit's place digit can be selected in 9 ways.

10's place digit can be selected in 9 ways.

100's place digit is 7

∴ it can be selected in 1 way.

∴ total numbers which have 7 in 100's place = 9 x 9 x1 = 81

∴ total number of numbers between 100 and 1000 having digit 7 exactly once

= 72 + 72 + 81 = 225.

11. How many four digit numbers will not exceed 7432 if they are formed using the digits 2, 3, 4, 7 without repetition?

Solution:

Among any set's of digits, greatest number is possible when digits are arranged in descending order.

7432 is the greatest number, formed from the digits 2, 3, 4, 7.

Since a 4-digit number is to be formed from the digits 2, 3, 4, 7, where repetition of digit is not allowed.

∴ 1000's place digit can be selected in 4 ways.

100's place digit can be selected in 3 ways.

10's place digit can be selected in 2 ways.

Unit's place digit can be selected in 1 way.

Total number of numbers not exceeding 7432 that can be formed from the digits 2, 3, 4, 7

= Total number of four digit numbers formed from the digits 2, 3, 4, 7 $=4 \times 3 \times 2 \times 1=24$

12. If numbers are formed using digits 2, 3, 4, 5, 6 without repetition, how many of them will exceed 400?

Solution:

Case I: Three digit numbers with 4 occurring in hundred's place:

∴ 100's place digit can be selected in 1 way.

10's place can be filled by any one of the number 2, 3, 5, 6.

∴ 10's place digit can be selected in 4 ways.

Unit's place digit can be selected in 3 ways.

∴ Total number of numbers which have 4 in 100's place = 1 x 4 x 3=12

Case II: Three digit numbers more than 500 .

100's place digit can be selected in 2 ways.

10's place digit can be selected in 4 ways.

Unit's place digit can be selected in 3 ways.

∴ Total number of three digit numbers more than 500 = 2 x 4 x 3 = 24

Case III: Number of four digit numbers formed from 2, 3, 4, 5, 6

Since, repetition of digits is not allowed

∴ total four digit numbers formed =5 x 4 x 3 x 2=120

Case IV: Number of five digit numbers formed from 2, 3, 4, 5, 6

Since, repetition of digits is not allowed

∴ total five digit numbers formed = 5x4x3x2x 1 = 120

∴ total number of numbers that exceed 400 = 12 + 24 + 120 + 120 = 276

13. How many numbers formed with the digits 0, 1, 2, 5, 7, 8 will fall between 13 and 1000 if digits can be repeated?
Solution:
Case 1: 2-digit numbers more than 13, less than 20, formed from the digits 0, 1, 2, 5, 7, 8
Number of such numbers — 3

Case II: 2-digit numbers more than 20 formed from 0, 1, 2, 5, 7, 8
Ten's place digit is selected from 2, 5, 7, 8.
∴ Ten's place digit can be selected in 4 ways.
Unit's place digit is any one from 0, 1, 2, 5, 7, 8
∴ Unit's place digit can be selected in 6 ways.
Using multiplication principle, the number of such numbers (repetition allowed) = 4 x 6 = 24

Case III: 3-digit numbers formed from 0, 1, 2, 5, 7, 8
100's place digit is any one from 1, 2, 5, 7, 8.
100's place digit can be selected in 5 ways.
As digits can be repeated, the 10's place and unit's place digits are selected from 0, 1, 2. 7, 8.
∴ 10's place and unit's place digits can be selected in 6 ways each.
Using multiplication principle, the number of such numbers (repetition allowed) = 5 x 6 x 6 = 180.
All cases are mutually exclusive and exhaustive.
∴ Required number = 3 + 24 + 180 = 207.

14. A school has three gates and four staircases from the first floor to the second floor. How many ways does a student have to go from outside the school to his classroom on the second floor?

Solution:
A student can go inside the school from outside in 3 ways and from first floor to second floor in 4 ways.

∴ Number of ways to choose gates = 3

∴ Number of ways to choose staircase = 4

By using fundamental principle of multiplication, number of ways in which student has to go from outside the school to his classroom = 4 x 3 = 12.

15. How many five-digit numbers formed using the digit 0, 1, 2, 3, 4, 5 are divisible by 3 if digits are not repeated?

Solution:

For a number to be divisible by 3.

The sum of digits must be divisible by 3.

Given 6 digits are 0, 1, 2, 3, 4, 5.

Sum of 1, 2, 3, 4, 5 = 15, which is divisible by 3.

∴ There are two cases of 5 digit numbers formed from 0, 1, 2, 3, 4, 5 and divisible by 3.

Either 3 is selected in 5 digits (and 0 not selected) or 3 is not selected in 5 digits (and 0 is selected)

Case I: 3 is not selected (and 0 is selected) i.e., the digits are 0, 1, 2, 4, 5.

∴ 10000's place digit can be selected in. ways (as 0 cannot appear).

As digits are not repeated, 1000's place digi can be selected in 4 ways.

100's place digit can be selected in 3 ways.

10's place digit can be selected in 2 ways.

Unit's place digit can be selected in 1 way.

Using multiplication theorem, Number of 5-digit number formed from 0, 1. 2, 4, 5 (with no repetition of digits)

=4 x 4 x 3 x 2 x 1 = 96

Case II: 3 is selected (and 0 is not selected) i.e., 1, 2, 3, 4, 5

10000's place digit can be selected in 5 ways.

1000's place digit can be selected in 4 ways.

100's place digit can he selected in 3 ways.

10's place digit can he selected in 2 ways.

Unit's place digit can be selected in 1 way. Using multiplication theorem, Number of 5-digit numbers formed from 1, 2. 3, 4, 5 =5 x 4 x 3 x 2 x 1= 120

Both the cases are mutually exclusive and exhaustive. Required number = 96 + 120 = 216