

## EXERCISE 6.2

Evaluate: (i)  $8!$  (ii)  $6!$  (iii)  $8! - 6!$  (iv)  $(8 - 6)!$

Solution :

$$i. 8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320.$$

$$ii. 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

$$\begin{aligned} iii. 8! - 6! &= 8 \times 7 \times 6! - 6! \\ &= 6! (8 \times 7 - 1) \\ &= 6! (56 - 1) \\ &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 55 \\ &= 39,600 \end{aligned}$$

$$iv (8 - 6)! = 2! = 2 \times 1 = 2$$

2. Compute: i.  $\frac{12!}{6!}$     ii.  $\left(\frac{12}{6}\right)!$     iii.  $(3 \times 2)!$     iv)  $3! \times 2!$

Solution:

$$i) \frac{12!}{6!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6!} = 6,65,280$$

$$ii) \left(\frac{12}{6}\right)! = 2! = 2 \times 1 = 2$$

$$iii) (3 \times 2)! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

$$iv) 3! \times 2! = 3 \times 2 \times 1 \times 2 \times 1 = 12$$

3. Compute:

$$i. \frac{9!}{3! 6!} \quad ii. \frac{6! - 4!}{4!} \quad iii. \frac{8!}{6! - 4!} \quad iv. \frac{8!}{(6 - 4)!}$$

*Solution:*

$$i. \frac{9!}{3! 6!} = \frac{9 \times 8 \times 7 \times 6!}{(3 \times 2 \times 1) \times 6!} = 84$$

$$ii. \frac{6! - 4!}{4!} = \frac{6 \times 5 \times 4! - 4!}{4!} = \frac{4!(6 \times 5 - 1)}{4!} = 29$$

$$iii. \frac{8!}{6! - 4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{6 \times 5 \times 4! - 4!} = \frac{1680}{29}$$

$$iv. \frac{8!}{(6 - 4)!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!} = 20160$$

#### 4. Write in terms of factorials

i.  $5 \times 6 \times 7 \times 8 \times 9 \times 10$

ii.  $3 \times 6 \times 9 \times 12 \times 15$

iii.  $6 \times 7 \times 8 \times 9$

iv.  $5 \times 10 \times 15 \times 20 \times 25$

*Solution:*

i.  $5 \times 6 \times 7 \times 8 \times 9 \times 10 = 10 \times 9 \times 8 \times 7 \times 6 \times 5$

*Multiplying and Dividing by 4!, we get*

$$\begin{aligned} &= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!} \\ &= \frac{10 \times 9 \times 8 \times \overset{4!}{7} \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4!} \\ &= \frac{10!}{4!} \end{aligned}$$

ii.  $3 \times 6 \times 9 \times 12 \times 15 =$

$$\begin{aligned} &3 \times (3 \times 2) \times (3 \times 3) \times (3 \times 4) \times (3 \times 5) \\ &= (3^5)(5 \times 4 \times 3 \times 2 \times 1) \\ &= 3^5(5!) \end{aligned}$$

iii.  $6 \times 7 \times 8 \times 9 = 9 \times 8 \times 7 \times 6$

*Multiplying and Dividing by 5!, we get*

$$\begin{aligned} &= \frac{9 \times 8 \times 7 \times 6 \times 5!}{5!} \\ &= \frac{9!}{5!} \end{aligned}$$

iv.  $5 \times 10 \times 15 \times 20 \times 25$

$$\begin{aligned} &= (5 \times 1) \times (5 \times 2) \times (5 \times 3) \times (5 \times 4) \times (5 \times 5) \\ &= 5^5 \times (5!) \end{aligned}$$

5. Evaluate :  $\frac{n!}{r!(n-r)!}$  for (i)  $n = 8, r = 6$

Solution :

i) When  $n = 8, r = 6$

$$\frac{n!}{r!(n-r)!} = \frac{8!}{6!(8-6)!}$$

$$= \frac{8 \times 7 \times 6!}{6! \times 2!}$$

$$= \frac{8 \times 7}{2 \times 1}$$

$$= 28$$

(ii)  $n = 12, r = 12$

Solution :

ii) When  $n = 12, r = 12$

$$\frac{n!}{r!(n-r)!} = \frac{12!}{12!(12-12)!}$$

$$= \frac{12!}{12! \times 0!}$$

$$= 0!$$

$$= 1$$

6. Find n if,

$$i. \frac{n}{8!} = \frac{3}{6!} + \frac{1}{4!}$$

*Solution:*

$$\begin{aligned} i. \frac{n}{8!} &= \frac{3}{6!} + \frac{1}{4!} \\ \therefore \frac{n}{8!} &= \frac{3}{6!} + \frac{6 \times 5}{6 \times 5 \times 4!} \\ \therefore \frac{n}{8!} &= \frac{3}{6!} + \frac{30}{6!} \\ \therefore \frac{n}{8 \times 7 \times 6!} &= \frac{33}{6!} \\ \therefore \frac{n}{56} &= 33 \\ \therefore n &= 56 \times 33 = 1848 \end{aligned}$$

$$ii. \frac{n}{6!} = \frac{4}{8!} + \frac{3}{6!}$$

*Solution:*

$$\begin{aligned} i. \frac{n}{6!} &= \frac{4}{8!} + \frac{3}{6!} \\ \therefore \frac{n}{6!} - \frac{3}{6!} &= \frac{4}{8!} \\ \therefore \frac{n - 3!}{6!} &= \frac{4}{8 \times 7 \times 6!} \\ \therefore n - 3 &= \frac{4}{8 \times 7} \\ \therefore n - 3 &= \frac{1}{14} \\ \therefore n &= \frac{1}{14} + 3 \\ \therefore n &= \frac{43}{14} \end{aligned}$$

$$iii. \frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$$

*Solution:*

$$\begin{aligned} iii. \frac{1}{n!} &= \frac{1}{4!} - \frac{4}{5!} \\ \therefore \frac{1}{n!} &= \frac{5}{5 \times 4!} - \frac{4}{5!} \\ \therefore \frac{1}{n!} &= \frac{1}{5!} - \frac{4}{5!} \\ \therefore \frac{1}{n!} &= \frac{1 - 4}{5!} \\ \therefore \frac{1}{n!} &= \frac{-3}{5!} \\ \therefore n &= 5 \end{aligned}$$

7. Find  $n$ , if

i.  $(n + 1)! = 42 \times (n - 1)!$

*Solution:*

$$(n + 1)! = 42(n - 1)$$

$$\therefore (n + 1)n(n - 1)! = 42(n - 1)!$$

$$\therefore n^2 + n = 42$$

$$\therefore n(n + 1) = 6 \times 7$$

Comparing on both side, we get

$$\therefore n = 6$$

ii.  $(n + 3)! = 110 \times (n + 1)!$

*Solution:*

$$(n + 3)! = 110 \times (n - 1)!$$

$$\therefore (n + 3)(n + 2)(n + 1)!$$

$$= 110(n + 1)!$$

$$\therefore (n + 3)(n + 2) = (11)(10)$$

Comparing on both side, we get

$$\therefore n + 3 = 11$$

$$\therefore n = 8$$

8. Find  $n$ , if :

$$i. \frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5:3$$

Solution:

$$i. \frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5:3$$

$$\therefore \frac{n!}{3!(n-3)!} \times \frac{5!(n-5)!}{n!} = \frac{5}{3}$$

$$\therefore \frac{n!}{3!(n-3)(n-4)(n-5)!} \times \frac{5 \times 4 \times 3!(n-5)!}{n!} = \frac{5}{3}$$

$$\therefore \frac{5 \times 4}{(n-3)(n-4)} = \frac{5}{3}$$

$$\therefore 12 = (n-3)(n-4)$$

$$\therefore (n-3)(n-4) = 4 \times 3$$

Comparing on both, we get

$$n-3 = 4$$

$$\therefore n = 7$$

$$ii. \frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!} = 10:3$$

Solution:

$$ii. \frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!} = 10:3$$

$$\therefore \frac{n!}{3!(n-5)!} \times \frac{5!(n-7)!}{n!} = \frac{10}{3}$$

$$\therefore \frac{n!}{3!(n-5)(n-6)(n-7)!} \times \frac{5 \times 4 \times 3!(n-7)!}{n!} = \frac{10}{3}$$

$$\therefore \frac{5 \times 4}{(n-5)(n-6)} = \frac{10}{3}$$

$$\therefore (n-5)(n-6) = 3 \times 2$$

Comparing on both, we get

$$n-5 = 3$$

$$\therefore n = 8$$



9. Find  $n$ , if:

$$i. \frac{(17-n)!}{(14-n)!} = 5!$$

Solution:

$$i. \frac{(17-n)!}{(14-n)!} = 5!$$

$$\therefore \frac{(17-n)(16-n)(15-n)(14-n)!}{(14-n)!}$$

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$\therefore (17-n)(16-n)(15-n) = 6 \times 5 \times 4$$

Comparing on both sides, we get

$$17-n = 6$$

$$\therefore n = 11$$

$$ii. \frac{(15-n)!}{(13-n)!} = 12$$

Solution:

$$ii. \frac{(15-n)!}{(13-n)!} = 12$$

$$\therefore \frac{(15-n)(14-n)(13-n)!}{(13-n)!} = 12$$

$$\therefore (15-n)(14-n) = 4 \times 3$$

Comparing on both sides, we get

$$15-n = 4$$

$$\therefore n = 11$$

10. Find n if

$$\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$$

Solution :

$$\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$$

$$\therefore \frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$$

$$\therefore \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)(2n-7)!}{7 \times 6 \times 5 \times 4! (2n-7)!} \times \frac{4!(n-4)!}{n(n-1)(n-2)(n-3)(n-4)!} = 24$$

$$\therefore \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)}{7 \times 6 \times 5} \times \frac{1}{n(n-1)(n-2)(n-3)} = 24$$

$$\therefore \frac{16(2n-1)(2n-3)(2n-5)}{7 \times 6 \times 5} = 24$$

$$\therefore (2n-1)(2n-3)(2n-5) = \frac{24 \times 7 \times 6 \times 5}{16}$$

$$\therefore (2n-1)(2n-3)(2n-5) = 9 \times 7 \times 5$$

Comparing both the sides, we get

$$2n-1 = 9$$

$$\therefore n = 5$$

11. Show that : (Question has been modified)

$$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$$

Solution:

$$L.H.S = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!} \left[ \frac{1}{r} + \frac{1}{n-r+1} \right]$$

$$= \frac{n!}{(r-1)!(n-r)!} \left[ \frac{n-r+1+r}{r(n-r+1)} \right]$$

$$= \frac{(n+1)n!}{r(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!(n-r+1)!}$$

$$= R.H.S$$

12. Show that :

$$\frac{9!}{3!6!} + \frac{9!}{4!5!} = \frac{10!}{4!6!}$$

Solution:

$$\text{L.H.S} = \frac{9!}{3!6!} + \frac{9!}{4!5!}$$

$$= \frac{9!}{3! \times 6 \times 5!} + \frac{9!}{4 \times 3! \times 5!}$$

$$= \frac{9!}{3!5!} \left[ \frac{1}{6} + \frac{1}{4} \right]$$

$$= \frac{9!}{3!5!} \left[ \frac{4+6}{6 \times 4} \right]$$

$$= \frac{10 \times 9!}{6 \times 5! \times 4 \times 3!}$$

$$= \frac{10!}{6!4!} = \text{R.H.S}$$

13. Find the value of :

i.  $\frac{8! + 5(4!)}{4! - 12}$

Solution:

i.  $\frac{8! + 5(4!)}{4! - 12}$

$$= \frac{8! + 5!}{4 \times 3 \times 2 - 12}$$

$$= \frac{8 \times 7 \times 6 \times 5! + 5!}{24 - 12}$$

$$= \frac{5!(8 \times 7 \times 6 + 1)}{12}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1(336 + 1)}{12}$$

$$= 5 \times 2 \times 337$$

$$= 3370$$

ii.  $\frac{5(26!) + (27!)}{4(27!) - 8(26!)}$

Solution:

ii.  $\frac{5(26!) + (27!)}{4(27!) - 8(26!)}$

$$= \frac{5(26!) + (27 \times 26!)}{4(27 \times 26!) - 8(26!)}$$

$$= \frac{26!(5 + 27)}{4(26!)(27 - 2)}$$

$$= \frac{32}{(4)(25)}$$

$$= \frac{8}{25}$$

14. Show that

$$\frac{(2n)!}{n!} = 2^n(2n-1)(2n-3) \dots 5.3.1$$

Solution:

$$\begin{aligned} \text{L.H.S} &= \frac{(2n)!}{n!} \\ &= \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4) \dots 6 \times 5 \times 4 \times 3 \times 2 \times 1}{n!} \\ &= \frac{(2n)(2n-1)[2(n-1)](2n-3)[2(n-2)] \dots (2 \times 3) \times 5 \times (2 \times 2) \times 3 \times (2 \times 1) \times 1}{n!} \\ &= \frac{2^n [n(n-1)(n-2) \dots 3.2.1] [(2n-1)(2n-3) \dots 5.3.1]}{n!} \\ &= \frac{2^n (n!)(2n-1)(2n-3) \dots 5.3.1}{n!} \\ &= 2^n(2n-1)(2n-3) \dots 5.3.1 \\ &= \text{R.H.S} \end{aligned}$$