Exercise 6.4

1 Find the number of permutations of letters in each of the following words:

i. DIVYA ii. SHANTARAM iii. REPRESENT iv. COMBINE

Solution:

i. There are 5 letters in the word DIVYA which can be arranged in 5! Way = 120 ways

ii. There are 9 letters in the word SHANTARAM in which 'A' repeats 3 times.

∴ Number of permutations of the letters of the word SHANTARAM

$$= \frac{9!}{3!} = 9 \; x \; 8 \; x \; 7 \; x \; 6 \; x \; 5 \; x \; 4 = 60480$$

iii. There are 9 letters in the word REPRESENT in which 'E' repeats 3 times and 'R' repeats 2 times. Number of permutations of the letters of the word REPRESENT

$$= \frac{9!}{3!\,2!} = \frac{9 \; x \; 8 \; x \; 7 \; x \; 6 \; x \; 5 \; x \; 4}{2} = 30240$$

iv. There are 7 distinct letters in the word COMBINE which can be arranged among themselves in 7! = 5040 ways

2. You have 2 identical hooks on English, 3 identical hooks on Hindi and 4 identical hooks on Mathematics. Find the number of distinct ways of arranging them on a shelf.

Solution:

There are total 9 books to be arranged on a shelf.

Out of these 9 books, 2 books on English, 3 books on Hindi and 4 books on Mathematics are identical.

∴ Total number of arrangements

$$= \frac{9!}{2!\,3!\,4!} = \frac{9x8x7x6x5x4!}{2x3x2x4!} = 9\ x\ 4\ x\ 7\ x\ 5\ =\ 1260$$

∴ In 1260 distinct ways the books can be arranged on a shelf.

3. A coin is tossed 8 times. In how many ways can we obtain
i. 4 heads and 4 tails?
ii. at least 6 heads?

Solution: A coin is tossed 8 times. All heads are identical and all tails are identical.

i. We can obtain 4 heads and 4 tails in $\frac{8!}{4!4!} = \frac{8x7x6x5}{4x3x2} = 70\ ways.$

∴ In 70 different ways we can obtain 4 heads and 4 tails.

ii. When at least 6 heads are to be obtained Outcome can be (6 heads and 2 tails) or (7 heads and 1 tail) or (8 heads)

∴ Number of ways in which it can be obtained

$$= \frac{8!}{6!\,2!} + \frac{8!}{7!\,1!} + \frac{8!}{8!} = \frac{8x7}{2} + 8 + 1 = 28 + 8 + 1 = 37\ Ways.$$

∴ In 37 different ways we can obtain at least 6 heads.

4. A bag has 5 red, 4 blue and 4 green marbles. If all are drawn one by one and their colours are recorded, how many different arrangements can be found?

Solution:

There are total 13 marbles in a bag.

Out of these 5 are Red, 4 Blue and 4 are Green: marbles. All balls of same colour are taken to be identical.

$\therefore$ Required number of arrangements $= \frac{13!}{5!4!4!}$

5.Find the number of ways of arranging letters of the word MATHEMATICAL. How many of these arrangements have all vowels together?

Solution:

There are 12 letters in the word MATHEMATICAL in which 'M' repeats 2 . times, 'A' repeats 3 times and 'T' repeats 2 times.

$\therefore$ Total number of arrangements $= \frac{12!}{2!3!2!}$

When all the vowels i.e., 'A', 'A', 'A', `E', 'I' are to be kept together Number of arrangements of these vowels $= \frac{5!}{3!}$ ways.

Let us consider these vowels together as one unit. This unit is to be arranged with 7 other letters in which 'M' and 'T' repeated 2 times each.

$\therefore$ Number of arrangements $= \frac{8!}{2!2!}$

$\therefore$ Total number of arrangements $= \frac{8!5!}{2!2!3!}$

Ex 6.4 : 6. Find the number of different arrangements of letters in the word MAHARASHTRA.

How many of these arrangements have

i. letters M and T never together?

ii. all vowels together?

Solution:

There are 11 letters in the word MAHARASHTRA in which 'A' is repeated 4 times, 'H' repeated 2 times and ‘R’ repeated 2 times.

∴ Total number of arrangements is $\frac{11!}{4!\,2!\,2!}$,

∴ $\frac{11!}{4!\,2!\,2!}$ different words can be formed from the letters of the word MAHARASHTRA.

i. Other than M and T. there are 9 letters in which A repeats 4 nines, H repeats twice, R repeats twice

The number of arrangements of the a letter = $\frac{9!}{4!2!2!}$ These 9 letters create 10 gaps in which M and T are to be arranged

The number of arrangement of M and T = ${}^{10}P_2$

∴ Total number arrangement having M and T never together = $\frac{9! \times {}^{10}P_2}{4!\,2!\,2!}$

ii. When all vowels are together.

There are 4 Vowels in the word MAHARASHTRA i.e. A, A, A, A

Let us consider these 4 vowels as one unit, they themselves can be arranged in $\frac{4!}{4!} = 1 way$.

This unit is to arranged with 7 other letters which can be done in 8! Ways

$\therefore$ Total number of arrangements $= \frac{8!}{2!2!}$

$\therefore \frac{8!}{2!2!}$ different words can be formed if vowels are always together.

7. How many different words are formed if the letter R is used thrice and letters S and T are used twice each?

Solution:

When 'R' is used thrice, 'S' is used twice and 'T' is used twice,

∴ Total number of letters available = 7, of which `S' and 'T' repeat 2 times each, 'R' repeats 3 times.

∴ Required number of arrangements

$$= \frac{7!}{2!2!3!} = \frac{7x6x5x4x3!}{2x1x2x1x3!} = 7x6x5 = 210$$

∴ 210 different words can be formed with the letter R is used thrice and letters S and T are used twice each.

8. Find the number of arrangements of letters in the word MUMBAI so that the letter B is always next to A.

Solution:
There are 6 letters in the word MUMBAI.
These letters are to be arranged in such a way that 'B' is always next to 'A'.
Let us consider AB as one unit. This unit with other 4 letters in which 'M' repeats twice, is to be arranged.
∴ Total number of arrangements when B is always next to A.

$$= \frac{5!}{2!} = \frac{5x4x3x2!}{2!} = 60$$

9 Find the number of arrangements of letters in the word CONSTITUTION that begin and end with N.

Solution:

There are 12 letters in the word CONSTITUTION, in which ‘O', 'N', 'I' repeat two times each, 'T' repeats 3 times.

The arrangement starts and ends with 'N', 10 letters other than N can he arranged between two N, in which ‘O' and 'I' repeat twice each and ‘T' repeats 3 times.

$\therefore$ Total number of arrangements with the letter N at the beginning and at the end $= \frac{10!}{2!2!3!}$.

10. Find the number of different ways of arranging letters in the word ARRANGE. How many of these arrangements have two 'R' and two A's not together?

Solution:

i. There are 7 letters in the word ARRANGE in which A is repeated 2 times and R is repeated 2 times

∴ The number of arrangements $= \frac{7!}{2!2!} = 1260$

ii. A: set of words having 2A together

B: set of words having 2R together

Number of words having both A and both R not together

$= 1260 - n(A \cup B)$

$= 1260 - [n(a) + n(B) - n(A \cap B)]$ ... (i)

n(A) = number of ways in which (AA) R, R, N, G, E are to be arranged

∴ $n(A) = \frac{6!}{2!} = 360$

n(B) = number of ways in which (RR), A, A, N, G, E are to be arranged

∴ $n(B) = \frac{6!}{2!} = 360$

$n(A \cap B)$ = number of ways in which (AA), (RR), N, G, E are to be arranged

∴ n(A n B) = 5! = 120

Substituting n(A), n(B), $n(A \cap B)$ in (i), we get

Number of words having both A and both R not together

= 1260 — [360 + 360 — 120]

= 1260 - 600

= 660

11, How many distinct 5 digit numbin, call be formed using the digits 3, 2, 3, 2, 4, 5.

Solution:

5 digit numbers are to be formed from 2, 3, 2, 3, 4, 5.

Case I: Numbers formed from 2, 2, 3, 4, 5 OR 2, 3, 3, 4, 5

Number of such numbers = $\frac{5!}{2!}\ x\ 2 = 5! = 120.$

Case II: Numbers formed from 2, 2, 3, 3 and any one of 4 or 5

Number of such numbers = $\frac{5!}{2!2!}\ x\ 2 = 60$

Required number = 120 + 60 = 180

∴ 180 distinct 5 digit numbers can be formed using the digit 3, 2, 3, 2, 4. 5.

12. Find the number of distinct numbers formed using the digits 3, 4, 5, 6, 7, 8, 9, so that odd positions are occupied by odd digits.

Solution:

A number is to be formed with digits 3, 4, 5, 6, 7, 8, 9 such that odd digits always occupy the odd places.

There are 4 odd digits i.e. 3, 5, 7, 9.

They can be arranged at 4 odd places among themselves in 4! ways = 24 ways

3 even places of the number are occupied by even digits (i.e. 4, 6, 8).

∴ They can be arranged in 3! ways = 6 ways

∴ Total number of arrangements =24 x 6=144

∴ 144 numbers can be formed so that odd digits always occupy the odd positions.

13. How many different 6-digit numbers can be formed using digits in the number 659942? How many of them are divisible by 2?

Solution:

A 6-digit number is to be formed using digits of 659942, in which 9 repeats twice.

$\therefore$ Total number of arrangements $= \frac{6!}{2!} = \frac{6X5X4X3X2!}{2!} = 360$

360 different 6-digit numbers can be formed.

For a number to be divisible by 2,

Last digits can be selected in 3 ways

Remaining 5 digit in which, 9 appears twice are arranged in ways $\frac{5!}{2!}$ Ways

$\therefore$ Total number of arrangements $= \frac{5!}{2!} x3 = 180$

$\therefore$ 180 numbers are divisible by 2.

14. Find the number of distinct word fromed from letters in the word INDIAN How many of them have the two N's together ?

Solution:

There are 6 letters in the word INDIAN in which I and N repeats twice.

Number of different words that can he formed using the letters of the word INDIAN

$$= \frac{6!}{2!\,2!} = \frac{6x5x4x3x2!}{2x2!} = 180$$

∴ 180 different words can be formed with the letters of word INDIAN.

i. When two N's are together.

Let us consider the two N's as one unit.

They can be arranged with 4 other letters in $\frac{5!}{2!} = \frac{5x4x3x2!}{2!} = 60\, ways$.

∴ 2 N can be arranged in 1 way

∴ Total number of arrangements = 60 x 1 = 60 ways

∴ 60 words are such that two N's are together.

15. Find the number of different ways of arranging letters in the word PLATOON if
i. the two O's are never together.
ii. consonants and vowels occupy alternate positions.

Solution:

i. When the two O's are never together:

Let us arrange the other 5 letters first, which can be done in 5! = 120 ways.

The letters P, L, A, T, N create 6 gaps. in which O's are arranged.

∴ Two O's in 6 gaps can be arranged in $\frac{{}^6P_2}{2!}$ $Ways$.

$= \frac{\frac{6!}{(6-2)!}}{2!} Ways$

$= \frac{6x5x4!}{4!\,x2x1} Ways$

$= 3\ x\ 5\ Ways$

$= 15\ Ways$

∴ Total number of arrangements if the two O's are never together = 120 x 15 = 1,800

ii. When consonants and vowels occupy alternate positions:

There are 4 consonants and 3 vowels in the word PLATOON.

∴ At odd places consonants occur and at even places vowels occur.

4 consonants can be arranged among themselves in 4! ways

3 vowels in which O occurs twice and A occurs once.

∴ They can be arranged in $\frac{3!}{2!}$ $Ways$

∴ Required number of arrangements if the Consonant and Vowels occupy alternate positions.

$= 4!\,x\frac{3!}{2!} = 4\ x\ 3\ x\ 2\ x\frac{3x2!}{2!} = 72$